APPLICATION OF CAPILLARY TUBES TO
LIQUID REFRIGERANT CONTROL

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May 20, 1949

Professor J. S. Newell
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Dear Sir:

In accordance with the requirements for the degree of Naval Engineer, we submit (with Jack B. Chaddock, Department of Mechanical Engineering) herewith a thesis entitled, "Application of Capillary Tubes to Liquid Refrigerant Control".

Respectfully,

APPLICATION OF CAPILLARY TUBES TO LIQUID REFRIGERANT CONTROL

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SUMMARY

Introduction

In typical refrigerating systems, an expansion valve is the device used to meter the flow of refrigerant. The capillary tube possesses certain advantages over the expansion valve, resulting in a considerable saving in manufacturing cost. The capillary is not rated in conventional terms, however, and as a consequence a cut-and-try process of design has been used.

The object of this thesis is a study of the flow process in the capillary tube, to permit a more straightforward approach to the design problem.

Procedure

The experimental apparatus was comprised of a typical open refrigerating system, with measuring instruments installed wherever specific data were needed, and control devices for setting the variables. All runs were made in essentially the same manner, differing only in the setting of these variables.

The first investigation was to determine the effect of two variables, namely, the pressure differential across the capillary, and the degree of subcooling. Next, dimensional analysis was applied to the capillary tube, in an attempt to determine the effect of other variables and simplify the test procedure. Finally, an investigation of the two flow processes in the capillary tube was made. These are the flow process from tube entrance to flash-off, and from flash-off to tube exit.

Results

The principal result was the discovery that the average friction factor, computed from the pressure drop due to friction and the average specific volume, was approximately that indicated from a plot of Reynolds number versus friction factor for drawn brass tubing. The parameter of Reynolds number used in this selection was computed from the liquid viscosity of Freon at the average refrigerant temperature maintained through the tube.

Conclusions

A method is presented for determining the correct length of an adiabatic capillary tube for use as a metering device in liquid refrigerant control This method is based on the conclusion that the friction factor is approximately constant over the entire length of the tube, and can be selected from a plot of Reynolds number versus friction factor for the appropriate tube roughness factor. The desired flow rate, end conditions, and friction factor for the capillary tubing to be used, must be selected. The correct length is then computed from the equations given in the thesis. A sample calculation is available in the Appendix.

Recommendations

The exact point of flash-off in the capillary tube should be determined more accurately. Therefore, it is recommended that a more accurate temperature or pressure measuing instrument be installed. In addition, the accuracy of data would be improved by the installation of devices to measure more accurately the power input, to obtain a check on the mass rate of flow, and to eliminate the slight variations

in discharge pressure in the present experimental apparatus.

For future investigation, a further study of the effect of increasing the pressure differential across the tube should be made. This could be followed by making tests with different lengths of tubes to determine the effect of L/D ratio, and to provide a check on the analysis presented for the six foot tube in this thesis.

INTRODUCTION

The capillary tube as a liquid metering device has become very popular within the last few years, especially since the advent of the Freon refrigerants. Today every manufacturer of domestic refrigerating systems is using, or is contemplating the use of capillaries to the exclusion of all devices previously used for metering the flow of liquid refrigerant, and the same is beginning to be true of smaller size commercial systems.

A capillary tube is a marvel of simplicity. Liquid refrigerant at high pressure flows into one end and expands down to evaporator pressure. The function of a capillary is to provide a restriction between the high and low pressure sides of the refrigerating system. In so doing, it meters refrigerant at the desired rate of flow. A capillary has no moving parts, nothing to wear out, and is no more difficult to install than a section of liquid line.

Another advantage offered by the capillary tube, in a refrigerating system using a hermetically sealed compressor, is that it allows the highside pressure to unload or balance out with the lowside pressure during the "off cycle", and thus permits the compressor to start in an unloaded condition. This unloading or balancing of pressures during the off cycle prevents the continuous storage of high pressure liquid, and hence effects a reduction in refrigerant requirements by reducing the gas charge to an absolute minimum. It is evident, therefore, that the use of a capillary will result in a considerable saving in manufacturing cost. Not only will the cost of an expansion valve, highside float, or other similar metering device be saved, but other

savings such as a reduced refrigerant charge, elimination of a liquid receiver, use of a less expensive motor, elimination of a compressor unloading device and simplification of the electrical system may likewise be effected. .

The field of application of capillaries so far has been restricted almost entirely to domestic refrigerators, and home freezers using factory assembled hermetic units. The reasons for restricting its use to such systems are that successful operation of the capillary requires:

1. The continuous maintenance of an accurate refrigerant charge over the entire life expectancy of the system. Since the gas charge is reduced to an absolute minimum, any leakage will soon make the system inoperative.
2. The maintenance of high standards of internal cleanliness and dehydration. Due to the small bore of the capillary it may be easily plugged by sludge, ice, etc.

Most engineers know, in a general way, how capillaries work. It is an obvious statement of fact that to increase flow, for example, it takes a larger bore, a shorter length, or a higher pressure differential across the tube. This statement just about tells the story of what capillaries are and how they work, but it does not give design information in specific terms by which capillaries can be analyzed, rated, or applied. For example, engineers in commercial or industrial practice can select a compressor for a certain Btu rating, coils to match the compressor which are rated in Btu per hour at a certain temperature difference can then be obtained. Then expansion valves, controls and various other accessories are selected. These are all

standard pieces of equipment that are carried in stock. Each carries its individual rating--in tons, Btu per hour, or horsepower. Capillaries, however, are not rated in conventional terms. In purchasing a capillary tube the engineer buys the material only, and it is up to him to make it work. If the tube fails to function properly, it can be shortened and another trial made. After several attempts at this cut and try process, he may be fortunate enough to find a workable capillary.

The purpose of this thesis, then, was a study of the flow process in the capillary tube, to permit a more straightforward approach to the problem of designing a workable capillary.

A capillary tube test set-up had been prepared in the Refrigeration Laboratory of the Massachusetts Institute of Technology by Sanford Klion and Edward Hanley. It consisted of a Chrysler Airtemp high speed condensing unit, with automatic discharge pressure control; a primary refrigerant calorimeter; and a 6 foot length, 0.055 inch diameter capillary. The capillary tube was completely insulated, and thermocouples mounted at short intervals along its length to study the temperature distribution. A complete description of this apparatus can be found in the Master of Science thesis by Klion and Hanley at the Massachusetts Institute of Technology.

In addition to the above equipment, an oil separator was added to the system to prevent the circulation of oil with the refrigerant through the capillary. Also a water bath heat exchanger with aquastat control was installed at the entrance to the capillary to provide a means of controlling the temperature of the refrigerant at this point.

PROCEDURE

Test Procedure

Testing a capillary tube involves control of the inlet conditions of the refrigerant and of the refrigerant rate of flow through the tube. Data must be obtained on the states through which the refrigerant passes as it traverses the tube. These states should be obtained at close intervals along the tube in order that a complete picture may be obtained of the process in the tube.

Control of the inlet conditions involves regulation of the pressure and temperature of the refrigerant as it enters the tube. Control of the refrigerant rate of flow for any tube of given length and diameter involves regulation of the pressure differential across the tube, as well as regulation of the subcooling at inlet. Some means must be provided for measurement of the refrigerant flow rate.

In the set-up for this thesis:

1. The inlet pressure was controlled by an automatic pressure regulator for the inlet water to the condenser.
2. The inlet temperature was controlled by a thermostatically controlled constant-temperature bath.
3. The outlet pressure was controlled to some extent by compressor speed and evaporator load.
4. The refrigerant flow rate was measured by a primary refrigerant calorimeter, and checked by the condenser heat transfer.
5. The inlet pressure was measured by pressure gage.

6. The states of the refrigerant along the tube were determined by temperature measurements with copper-constantan thermocouples.
7. The outlet pressure was measured by pressure gage.
8. The effects of heat transfer along the capillary tube proper were minimized by insulating the tube thoroughly with santocel.

<u>Thesis Procedure</u>

The original intent of this investigation was to find some method of making the application of capillary tubes to liquid refrigerant control a straightforward engineering problem to be solved by direct mathematical or empirical means, and preferably by unequivocal design charts requiring a minimum of knowledge of the complexities of capillary flow. The first consideration was given to the ascertainment of the effect of those variables thought to have the greatest effect in refrigerant control, and an attempt was made to delimit the capillary flow process entirely in terms of these variables. For the first runs the inlet pressure and the compressor speed were fixed, while the calorimeter load and refrigerant subcooling were allowed to vary. Straight design chart methods were then applied to the effects of pressure differential and refrigerant subcooling on the refrigerant rate of flow. Such methods quickly proved impractical, because of the obvious effect of other variables.

The next step was an attempt to apply dimensional analysis methods to the overall flow process. Conditions were allowed to vary with an intentional randomness. Dimensional analysis, however, could

not be applied satisfactorily because of the lack of a sufficient range of data.

Finally the investigation narrowed itself down to an examination of the flow process within the capillary tube itself. This examination was in two parts: the flow process up to the flash-off point; and the flow process after the flash-off point. When the trend of the investigation became apparent, pressure differential and subcooling conditions were controlled methodically, and as closely as the inherent limitations of the apparatus would permit. Several runs were made at a higher discharge pressure, to effect a considerable change in the pressure differential; and with different amounts of subcooling and calorimeter load, to obtain varying rates of refrigerant flow. The last three runs were made at the original discharge pressure, but with a much higher compressor speed.

Evaluation of Data

Despite the apparent adequacy of the number of thermocouples used on the tube, and despite the seeming logic of their spacing, the greatest difficulty encountered in this investigation was the determination of the exact point of flash-off for any one run. In particular, when subcooling was held to a small amount and flash-off took place nearer to the tube inlet than at other times, the wide spacing of the first few thermocouples necessitated considerable guessing at the location of flash-off. When subcooling approached the other extreme, the flash-off point could be located reasonably well, but the data on the flow process after flash-off became too sketchy for a good analysis.

One other drawback in the evaluation of data was the failure of the condenser check on refrigerant rate to agree as closely with

the calorimeter determination as should be expected from this type of heat balance.

RESULTS

1. The flow rate in a capillary tube increases approximately linearly with an increase in the degree of subcooling at the tube entrance, as shown in Figure 1.
2. The flow rate increases with increased pressure differential across the tube, also shown in Figure 1.
3. Figures 2, 3, and 4 show the temperature and pressure distribution along the capillary tube, when subcooled liquid enters the capillary. From tube entrance, at point 1, to flash-off, at point 2, the temperature is constant and the pressure drop linear. From point 2 to point 3, at the tube exit, the pressure and temperature drop is not linear.
4. The refrigerant is entirely in the liquid state from points 1 to 2, and the flow process follows familiar straight pipe equations of hydraulics. The friction factor and Reynolds number computed from these equations appear in the table of results.
5. From points 2 to 3, the percentage and volume of vapor increases in the direction of flow. The quality and specific volume of the vapor at the tube exit are tabulated in the table of results.
6. The flow process in section 2-3 of the tube is a compressible flow of liquid and vapor and follows the momentum equation. The friction factor and pressure drop due to momentum were calculated from this equation and may also be found in the table of results.
7. The pressure drop due to friction is from 2 to 3 times the pressure drop due to momentum change for the entire capillary, with subcooled liquid entering the tube.

8. The integrated average specific volume for the entire capillary was found to be roughly twice that for section 1-2, for the runs calculated.

9. The average friction factor, computed from the pressure drop due to friction and the average specific volume, was approximately that indicated from a plot of Reynolds number versus friction factor for drawn brass tubing. The parameter of Reynolds number was calculated from the liquid viscosity of Freon at the average refrigerant temperature through the capillary tube.

10. There is a definite drop in pressure and temperature from the tube exit to the evaporator. This drop was generally from 12 to 15 degrees Fahrenheit.

11. Due to the large number of variables involved in the flow process of the capillary, dimensional analysis was applied, but the data did not cover a wide enough range to show any significant trends of the dimensional numbers arrived at.

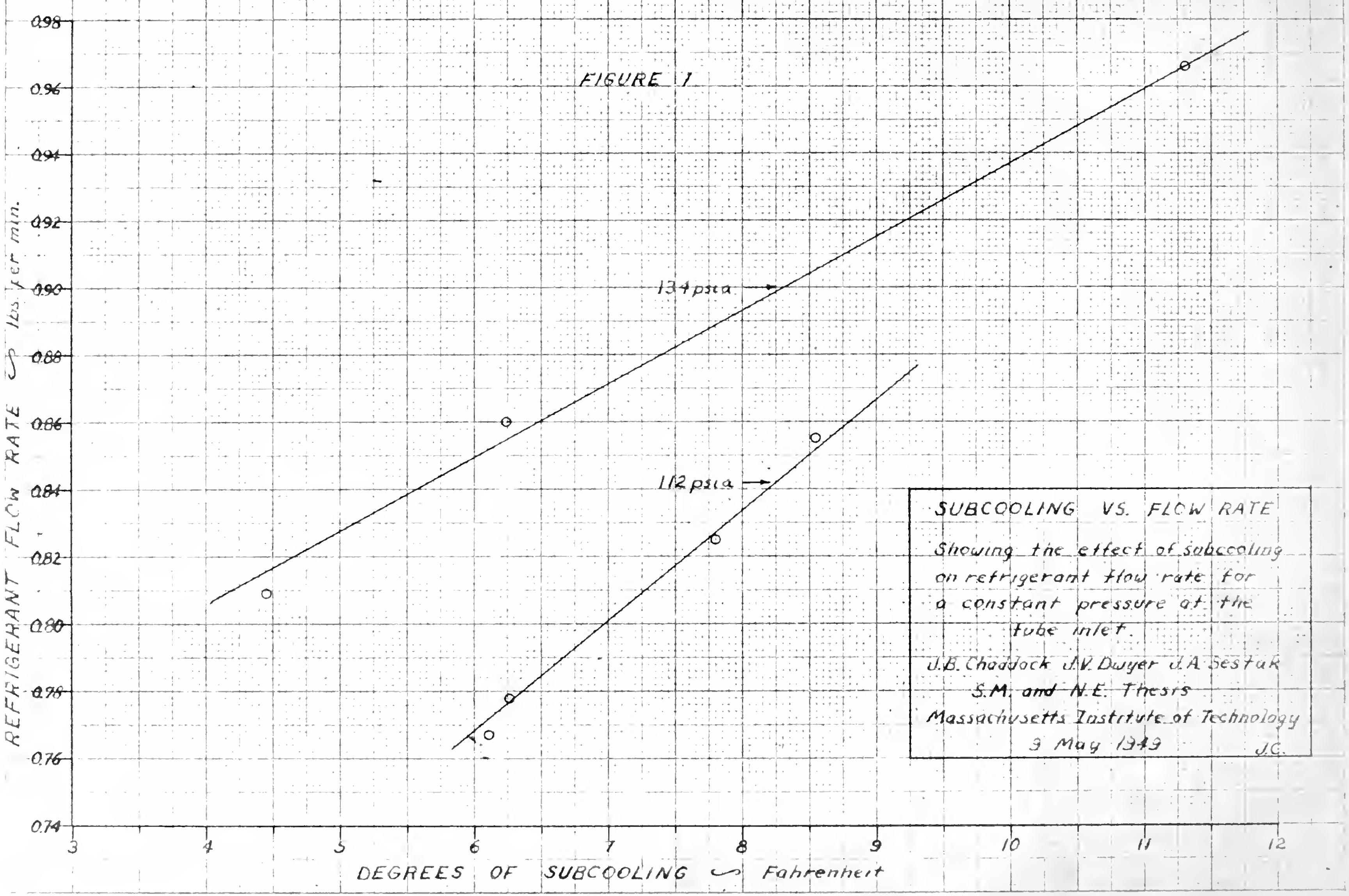
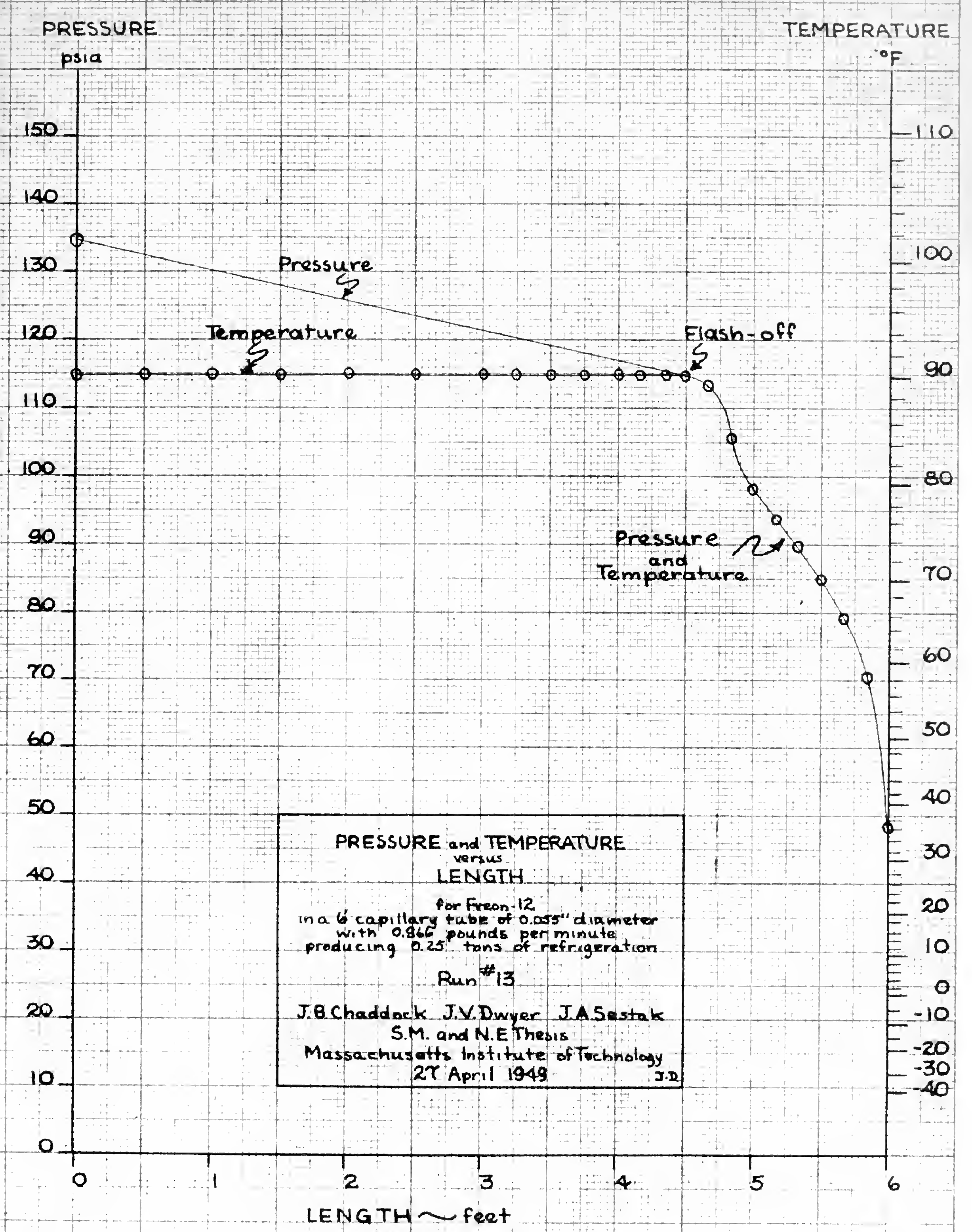


FIGURE 1
REFRIGERANT FLOW RATE ~ lbs per min.
0.98
0.96
0.94
0.92
0.90
0.88
0.86
0.84
0.82
0.80
0.78
0.76
0.74
3
4
5
6
7
8
9
10
11
12
DEGREES OF SUBCOOLING ~ Fahrenheit
134 psia
112 psia
SUBCOOLING VS. FLOW RATE
Showing the effect of subcooling on refrigerant flow rate for a constant pressure at the tube inlet.
J.B. Chaddock J.V. Dwyer J.A. Sestak
S.M. and N.E. Thesis
Massachusetts Institute of Technology
3 May 1949
J.C.

FIGURE 2


PRESSURE psia
TEMPERATURE °F
Pressure
Temperature
Flash-off
Pressure and Temperature
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
110
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
0
1
2
3
4
5
6
LENGTH ~ feet
PRESSURE and TEMPERATURE
versus
LENGTH
for Freon-12
in a 6' capillary tube of 0.055" diameter
with 0.866 pounds per minute
producing 0.25 tons of refrigeration
Run #13
J.B. Chaddock J.V. Dwyer J.A. Sestak
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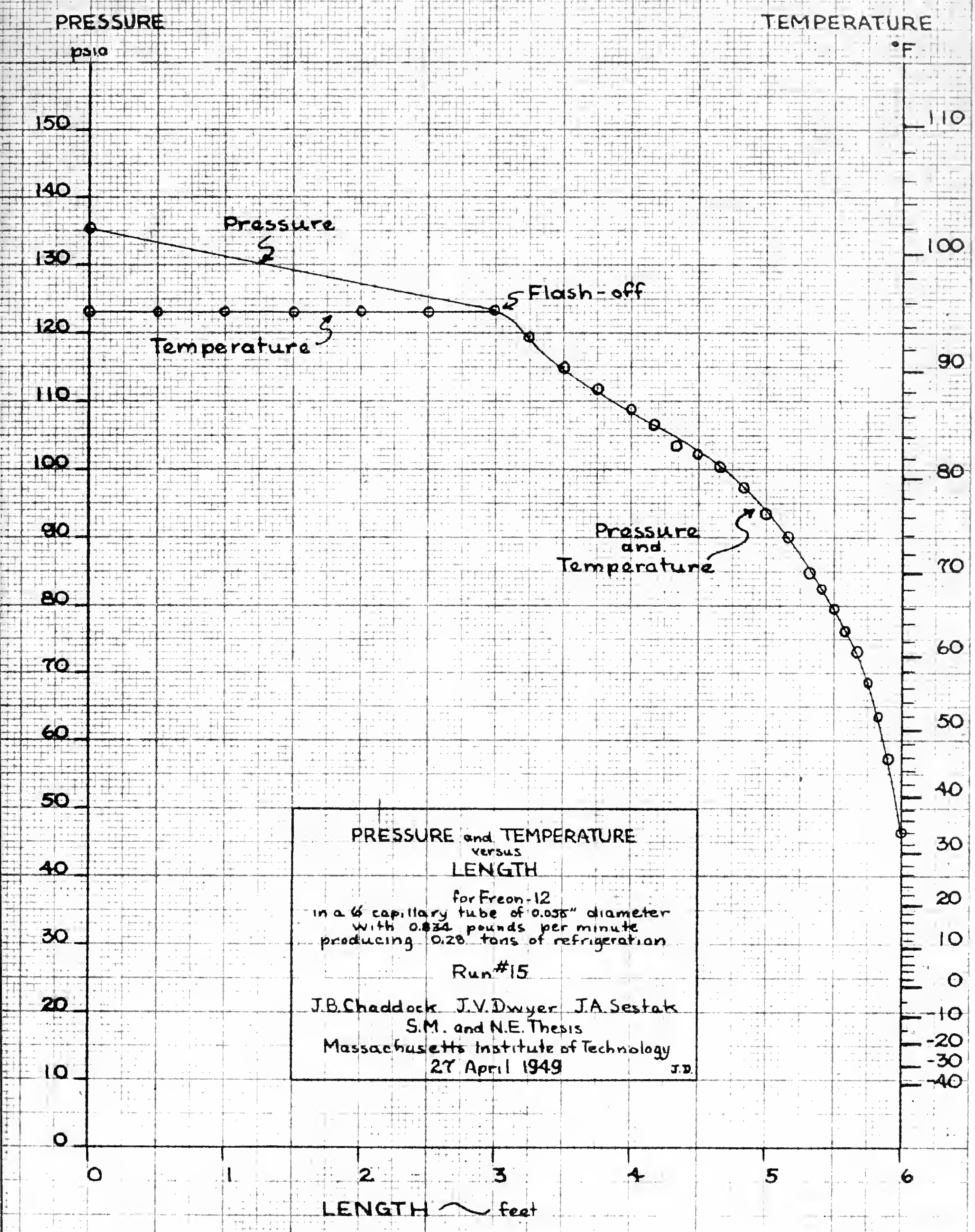
Massachusetts Institute of Technology
27 April 1949
J.D.

FIGURE 3



PRESSURE
psia
TEMPERATURE
°F
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
110
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
Pressure
Flash-off
Temperature
Pressure and Temperature
0
1
2
3
4
5
6
LENGTH ~ feet
PRESSURE and TEMPERATURE
versus
LENGTH
for Freon-12
in a 6' capillary tube of 0.036" diameter
with 0.834 pounds per minute
producing 0.28 tons of refrigeration
Run #15
J.B. Chaddock J.V. Dwyer J.A. Sestak
S.M. and N.E. Thesis
Massachusetts Institute of Technology
27 April 1949
J.D.

FIGURE 4

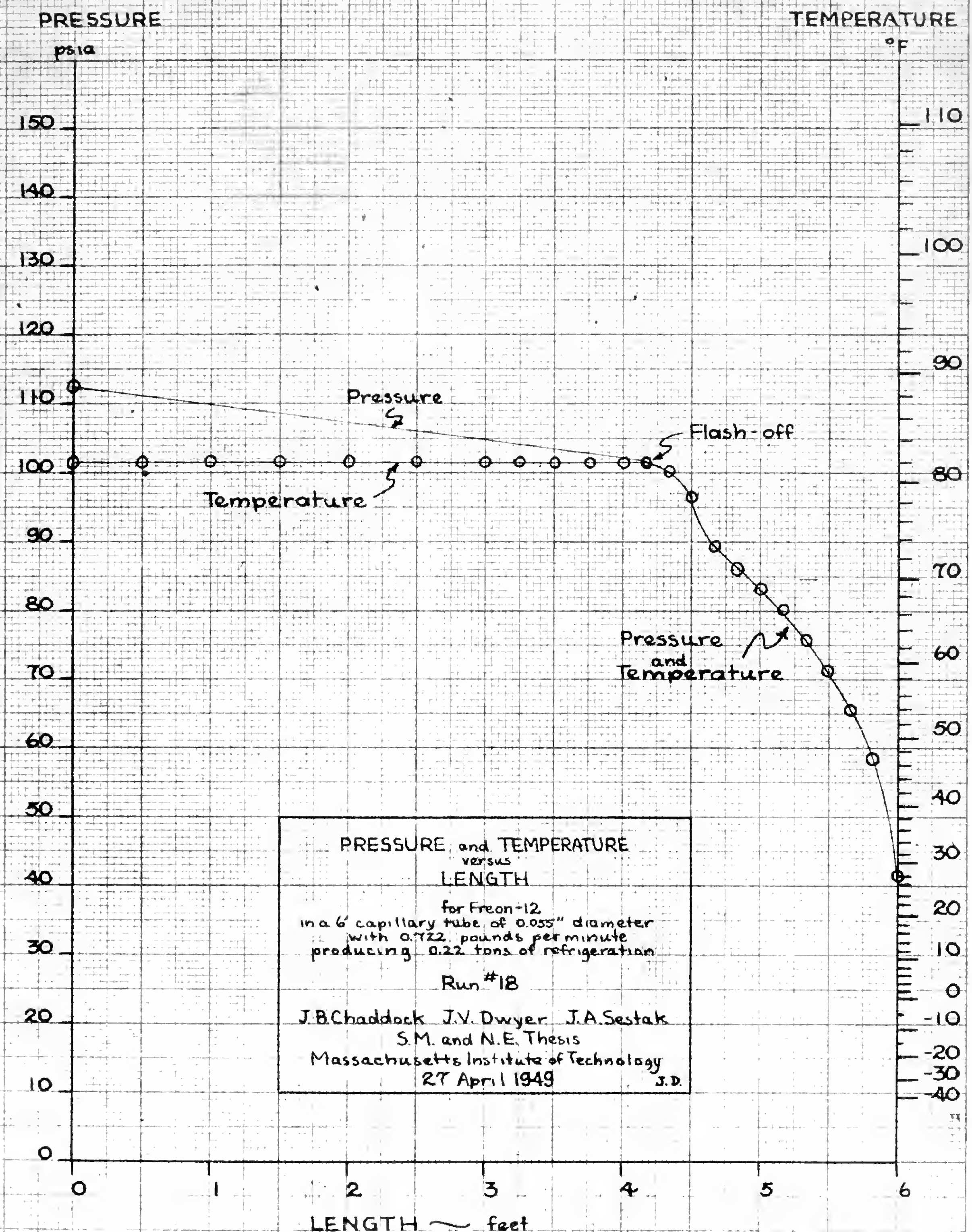

PRESSURE
psia
TEMPERATURE
°F
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
110
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
Pressure
Flash-off
Temperature
Pressure
and
Temperature
PRESSURE and TEMPERATURE
versus
LENGTH
for Freon-12
in a 6' capillary tube of 0.055" diameter
with 0.722 pounds per minute
producing 0.22 tons of refrigeration
Run #18
J.B. Chaddock J.V. Dwyer J.A. Sestak
S.M. and N.E. Thesis
Massachusetts Institute of Technology
27 April 1949
J.D.
0
1
2
3
4
5
6
LENGTH ~ feet

TABLE I

CAPILLARY TUBE RESULTS

Run No.	Degrees Subcooling	Evap. Load	Mass Rate of Flow	Mass Velocity	Tube Exit Temp.	Friction Factor	Reynolds Number	Flash-off Length	Quality	Specific Volume	Friction Factor	Pressure Drop in Tube: Total	Pressure Drop in Tube: Momentum	Pressure Drop in Tube: Friction	Specific Volume	Friction Factor
	t_s °F	Q BTU/HR.	w_r LB./MIN.	G LB./FT.² SEC.	t_3 °F	f_{1-2}	Re_{1-2}	L_{1-2} FT.	X_3	v_3 FT.³/LB.	f_{2-3}	Δp psi	$\Delta p_{mom.}$ psi	Δp_f psi	$v_{av.}$ FT.³/LB.	$f_{av.}$
							$\times 10^{+4}$								$\times 10^{-1}$	
2	6.11	3103	0.758	765.6	38.3	0.00311	2.06	4.50	0.150	0.132		60.6	15.1	45.5		
3	7.75	2465	0.760	767.6	28.0	0.00373	2.06	4.67	0.173	0.178		69.1	21.1	48.0		
4	7.46	3161	0.787	794.9	32.2	0.00349	2.13	4.50	0.161	0.156	0.00696	65.5	19.6	45.9	0.231	0.00556
5	6.33	3416	0.801	809.0	34.2	0.00288	2.18	4.50	0.163	0.152		64.7	19.8	44.9		
6	6.11	2686	0.767	774.7	31.2	0.00306	2.10	4.50	0.172	0.168		67.7	20.2	47.5		
8	6.26	2506	0.778	785.8	31.2	0.00304	2.13	4.50	0.172	0.168	0.00763	67.9	20.8	47.1	0.234	0.00576
9	6.82	2482	0.808	816.1	38.3	0.00404	2.28	4.00	0.193	0.167		83.1	22.2	60.9		
10	12.30	3400	0.941	950.4	39.3	0.00428	2.61	4.67	0.169	0.145		81.8	25.9	55.9		
11	10.84	2797	0.896	905.0	35.0	0.00442	2.49	4.50	0.184	0.168		85.2	27.6	57.6		
12	4.46	3027	0.809	817.1	34.7	0.00253	2.37	3.75	0.213	0.194		87.3	26.1	61.2		
13	11.30	3050	0.966	975.4	36.0	0.00391	2.70	4.50	0.184	0.165	0.00581	86.5	31.2	55.3	0.208	0.00495
14	6.23	3377	0.860	868.6	37.3	0.00304	2.49	3.75	0.200	0.175		85.9	26.4	59.5		
15	6.84	3377	0.834	842.4	34.0	0.00487	2.35	3.00	0.208	0.191	0.00555	89.0	27.4	61.6	0.284	0.00540
16	6.79	2476	0.808	815.8	34.7	0.00430	2.28	3.50	0.203	0.185	0.00693	87.7	24.8	62.9	0.268	0.00625
17	7.80	2926	0.825	833.2	29.8	0.00328	2.23	4.67	0.172	0.171	0.00737	69.9	23.8	46.1	0.213	0.00552
18	6.95	2613	0.772	780.0	27.5	0.00364	2.10	4.25	0.180	0.186	0.00752	71.1	22.8	48.3	0.232	0.00605
19	8.54	3428	0.855	863.2	31.3	0.00327	2.32	4.67	0.166	0.162	0.00666	69.1	24.0	45.1	0.210	0.00512

DISCUSSION OF RESULTS

The effect of subcooling for a given pressure entering the tube is seen to have a very marked effect on the flow rate. The increase in flow rate with subcooling over the range of values covered in the tests was approximately linear as shown in Figure 1 of the results. However, it is believed that this linear relationship applies over a limited range only, that is, at very high degrees of subcooling the increase in flow rate will not be as great as shown in these curves. The reason for this increase in flow rate with subcooling may be explained as follows. If liquid at saturation temperature enters the capillary it will begin to expand, forming flash gas right at the entrance, which increases the restriction to flow. If, however, the liquid is subcooled below its saturation temperature, it will travel for some distance through the tube before the flash gas starts to form, and, consequently, there will be less restriction to flow.

The effect of increasing the inlet pressure is also shown in Figure 1, the flow rate increasing with increased pressure differential across the tube. This is also well illustrated by reference to test runs No. 5 and 14 in the table of results. These runs were made at approximately the same number of degrees of subcooling and evaporator load, but run No. 14 was made with a 21 psi greater pressure differential. For this 32 per cent increase in pressure difference the flow rate increased by 0.058 pounds per minute, or 7 1/4 per cent.

With subcooled liquid entering the capillary, the pressure and temperature distribution is as shown in Figures 2, 3, and 4. These plots were made with a saturation temperature scale corresponding to

the pressure scale superimposed along the vertical axis, so that both could be represented on the one plot. From point 1, at the tube entrance, to point 2, at the flash-off, the pressure drop is linear and the temperature constant. In this portion of the tube the refrigerant is entirely in the liquid state, and at point 2 the first bubble of vapor forms. From point 2 to point 3, at the end of the tube, the pressure drop is not linear. After the initial "break" at the flash-off point, the pressure and temperature drop per unit length increases as the end of the tube is approached. For this portion of the tube both the saturated liquid and saturated vapor phases are present, so that the temperature and pressure are both represented by a single line on the plot. The percentage of vapor increasing in the direction of flow results in momentum changes which cause the non-linear variation in pressure in this section of the tube. The "break" referred to in these curves is the sudden pressure and temperature drop just after the flash-off at 2. There is no logical explanation for this sudden drop by analysis of the equations of flow. The momentum equation indicates that the pressure differential should increase from here to the end of the tube, which it does over the latter part of the curve, but does not account for the "break". Other experimenters, including Bolstad and Jordan, obtained smooth curves for this region, it is believed, therefore, that the curves are slightly in error at this point. This may be due to some heat transfer along the tube.

The adiabatic flow process in the capillary tube is really made up of two processes. The isothermal turbulent flow up to the flash-off point, and the flashing flow of liquid and vapor from there to the end of the tube. These two processes must be considered sepa-

rately in analyzing the flow through the capillary.

For the isothermal turbulent flow process in section 1-2 of the capillary where the refrigerant is entirely in the liquid state, the flow follows the well known straight pipe equations. The friction factor may be computed from the Fanning equation in the following form.

$$f_{1-2} = \frac{g\,D\,(p_1 - p_2)}{2\,L_{1-2}\,G^2\,v_1} \qquad (4)$$

The friction factors, thus computed, appear in the table of results. A rather wide variation in these values may be noted. It is believed that the reason for this variation is in the determination of the length to the flash-off point. Temperature and pressure length curves, similar to those shown in the results, were drawn for each test run to determine this quantity. In discussing these curves above, however, it was noted that they are slightly in error in this region. In addition, the thermocouples were spaced at from two to three inch intervals in the vicinity of the flash-off, and with the potentiometer available could not be read closer than a half a degree Fahrenheit. Therefore, the exact determination of this flash-off point was impossible.

The friction factor is known to be a function of the tube roughness and Reynolds number. The latter may be calculated for this section by:

$$Re_{1-2} = \frac{G\,D}{\mu_1} \qquad (6)$$

These values are also tabulated in the results, and show that the range covered was very small, all these numbers being between 2.0×10^4 and 2.7×10^4. Reference to a plot of Reynolds number versus friction factor

for drawn brass tubing shows that the value of friction factor for this range should be of the order of 0.006. The values calculated are considerably lower than this, which indicates that the flash-off occurred closer to the entrance of the tube than was found from the temperature and pressure versus length plots. Elimination of the "break" in these plots by drawing a smooth curve from the low pressure end back to the linear part of the pressure drop line over section 1-2 of the tube, as suggested above, would give a much better approximation to where the flash-off actually occurred.

The second process in the capillary is of a much more complicated nature, being a flashing flow of liquid and vapor from points 2 to 3. From the steady flow energy equation the following formula can be developed for finding the quality of vapor present at any point beyond the flash-off where the temperature is known. As written below it is between points 2 and 3, but it is equally valid between point 2 and any other point along the tube in the direction of flow.

$$x_3^2 + \left[\frac{2v_{f_3}}{v_{fg_3}} + \frac{2gJ\, h_{fg_3}}{G^2(v_{fg_3})^2}\right] x_3 - \frac{2gJ\,(h_{f_2} - h_{f_3})}{G^2\,(v_{fg_2})^2} = 0 \qquad (11)$$

The temperature at point 2 being the same as at the tube entrance for adiabatic flow in the capillary, a knowledge of the end conditions only are necessary to make this calculation. The quality at the tube exit was thus calculated for each test run. These values range from approximately 15 to 20 per cent.

With x_3 determined h_3, v_3, and s_3 may be readily found by using:

$$h_3 = h_{f_3} + x_3\, h_{fg_3}, \quad v_3 = v_{f_3} + x_3\, v_{fg_3}, \quad \text{etc.}$$

The specific volume at the tube exit was computed by this equation and appears in the table of results. If temperatures are known at several points between point 2 and 3, similar calculations can be carried out for these points and the thermodynamic state path of the two phase flow exactly determined.

The simple straight pipe equations for determining flow rate do not apply to this two phase flow. As mentioned above, pressure drop in this section of the tube is not caused by friction alone, but both by friction and by momentum changes in the fluid. The equation governing flow for this second section of the tube is the momentum equation, which takes account of both these factors. Its application between points 2 and 3 of the tube yields the following formula.

$$\frac{f_{2-3}}{r} \int_2^3 v\,d\,L = \frac{g}{G^2} (p_2 - p_3) - (v_3 - v_2) \qquad (15)$$

The integral on the left hand side of this equation must be evaluated graphically. That is, the quality is determined at short intervals along the tube between points 2 and 3 by means of equation (11). The corresponding specific volumes are then readily calculated and plotted against length from the flash-off point, and the area under the curve obtained by means of a planimeter. With the value of the integral thus determined, the friction factor for this section of the tube may be readily calculated. Reference to the results shows these values computed for several of the runs. A variation is again evident and the values are all high, which may again be explained by the flash-off point being taken too far from the tube entrance.

Having analyzed both sections of the tube and having found

equations which govern the flow rates, the entire tube was now studied in an attempt to bring these equations together. In this manner equations for designing capillary tubes were arrived at. Since the pressure drop in the tube is a combination of friction and momentum, the first step was to determine how much of the total pressure drop was due to momentum change. This was found by applying the momentum equation without the friction term.

$$\Delta p_m = \frac{G^2}{g} (v_3 - v_2) \tag{15a}$$

It may be noted here, also, that a knowledge of the end conditions only are necessary to find this quantity.

Subtracting this pressure drop due to momentum change from the total pressure differential across the tube gives the pressure drop which is due to friction.

$$\Delta p_f = \Delta p_t - \Delta p_m \tag{16}$$

These three pressure drops are also tabulated in the results. The pressure drop due to momentum was between 20 and 30 psi for all runs except No. 2.

Having constructed the plots of specific volume versus length for section 2-3 of the tube in order to evaluate the integral in equation (15), this same plot was used to find the average specific volume for this section. Since the specific volume in the first section is constant, the average specific volume for the capillary tube is given by:

$$v_{av.} = v_1 \left(\frac{L_{1-2}}{L}\right) + v_{2-3} \left(\frac{L_{2-3}}{L}\right) \tag{17}$$

Equation (4) may now be applied to find the average friction factor for the capillary.

$$f_{av.} = \frac{g\,D\Delta p_f}{2L\,G^2\,v_{av.}} \qquad (4a)$$

The average friction factor thus determined depends on the length to flash-off only to the extent of determining the average specific volume in the tube. It is only near the end of the tube, however, that the specific volume begins to increase appreciably. A considerable error can be made in determining where the flash-off occurs, therefore, with only a slight effect upon the average specific volume. This average friction factor, then, can be considered the most accurate of the friction factors calculated. The results show that the variation in this quantity is not too great, and is of the right order of magnitude for drawn brass tubing at the specific Reynolds number.

Since these average friction factors check closely, a method of designing capillary tubes may be arrived at by making the following assumption. The friction factor remains constant over the entire length of the tube, and is a function of the Reynolds number calculated from the liquid viscosity at the average temperature of the refrigerant in the tube.

There may be some question as to the justification of using the liquid viscosity for computation of Reynolds number for two phase flow, and thus the parameter of Reynolds number for such flow could be open to question. However, a study of the test results shows such an assumption to be not far from the truth. For example, considering any one of the test runs where the average friction factor was calcu-

lated, if this average friction factor be applied to section 1-2 of the tube the corresponding length to flash-off may be found from equation (4). This affects the calculations of section 2-3 only to the extent of increasing the area under the curve of specific volume versus length, thus increasing the value of the integral in equation (15). The end points 2 and 3 have not been affected. Increasing this integral naturally lowers the value of friction factor for section 2-3 of the tube, and in every case will bring it nearly into agreement with the average friction factor. This shows that the factor is approximately constant.

This assumption does not deny that gas is formed in the tube, but merely means that liquid refrigerant remains in contact with the capillary tube wall over the entire length of the tube. As stated above, it is only near the end of the tube that the volume of gas increases appreciably, so that from this standpoint, also, the assumption seems well justified.

To summarize, there are only two factors which would change the frictional effects in the last part of the tube: first, the drop in temperature, which increases the viscosity of the refrigerant; and second, contact of gas with the tube wall, which decreases the viscosity of the refrigerant. The first factor may be taken into consideration by using an average temperature in determining the viscosity; the second may be assumed negligible by reason of the discussion above.

Based on the foregoing, the following procedure may be followed to determine the correct length of capillary tubing for a given flow rate and set of entering and leaving conditions for the tube. It is well to note here that the evaporating temperature which

is to be held by the system is not the same as the temperature at the tube exit. The data observed show that there is a significant drop in pressure, and hence in temperature, from the tube exit to the evaporator. This temperature drop was approximately 12 F for an entering pressure of 110 psia, and 15 F for an entering pressure of 135 psia. The temperature at the tube exit may then be determined by adding the appropriate value to the evaporating temperature. With this temperature determined, the quality at the tube exit is calculated by equation (11), and the specific volume at this point is readily found. Equation (15a) may now be applied to find the pressure drop due to momentum, and subtraction of this quantity from the total pressure drop across the tube gives the pressure drop due to friction. The Reynolds number is calculated from the desired flow rate and viscosity of liquid Freon at the average refrigerant temperature. The friction factor may then be obtained from a plot of Reynolds number versus friction factor. The tube roughness for the capillary tubing used must be known to make this selection. In this thesis restrictor tubing was used, and the friction values checked closely with those for drawn brass tubing.

Next the average specific volume must be determined. This is the only difficult part of the calculation. To date, no equation has been worked out that will accurately determine this quantity. The curve obtained by plotting specific volume against length from flash-off may be found in the appendix for run No. 15. Similar curves were obtained for all the other runs for which the average friction factor was calculated. These curves were plotted on logarithmic paper to determine whether they following an exponential form of equation, but with no success. A rough value may be found from the following relation.

$$v_{av} = v_1 + \frac{v_3}{16}$$

This value checked against those obtained from the plots with no greater than 10 per cent error.

All the values in equation (4a) have now been selected or determined except the length and diameter of capillary tube. A standard bore capillary may then be selected and the required length computed. If this length is unsatisfactory for the particular application desired, the following formula developed by L. A. Staebler giving the relationship between bore and length for equivalent flow capacities may be used to find the length of other standard bore capillaries for the same flow rate.

$$L_1 = L_2 \left(\frac{D_1}{D_2}\right)^{4.6}$$

A derivation of the equations discussed in this section may be found in the discussion in the appendix. Also a complete set of calculations illustrating the use of these equations in determining the results, and in calculating the required length of capillary tubing may be found in the sample calculations in the appendix.

Because of the large number of variables involved in the capillary tube flow process it would require innumerable tests to determine the effect of these variables separately. For this reason dimensional analysis was applied to the problem to simplify the test procedure. Four dimensional numbers were determined, which are listed below.

$$Re = f\left(\frac{L}{D}, \frac{P_1}{P_1 - P_3}, \frac{P_1}{P_2}\right)$$

Upon first inspection it may appear that the important variable of subcooling is not included in this analysis. It is taken into account, however, by the ratio of p_1/p_2. With saturated liquid entering the tube this ratio is unity. As the degree of subcooling increases the ratio increases proportionally.

Since the time required for each test run was from five to six hours, insufficient runs were made to apply this analysis effectively. Only one tube was tested, which eliminated any variation in the L/D ratio. Also this limitation to one tube held the Reynolds number range covered to a small interval, as was discussed earlier in this section. Two discharge pressures were used, but again no wide variation was obtained. Therefore, the analysis could not be applied. However, it is believed that if sufficient data are obtained, the above analysis, or one similar to it, will aid in bringing all the data together to give a more complete understanding of the process.

CONCLUSIONS

1. The pressure and temperature distribution curves, Figures 2, 3, and 4, are slightly in error in the region of the indicated flash-off point. A smooth curve from the linear pressure line of section 1-2 to the end of the tube will give a much closer value of the true flash-off length.
2. This adjustment of flash-off length will bring the friction factors for both sections of the tube into very close agreement with each other, as well as with the average friction factor calculated for the entire capillary tube.
3. The average friction factor was not affected by the error in determination of flash-off length, and is considered an accurate result.
4. The friction factor, therefore, is approximately constant over the entire length of the tube, and can be selected from a plot of Reynolds number versus friction factor for the appropriate tube roughness factor. The parameter of Reynolds number used in this selection is to be computed from the liquid viscosity of Freon at the average temperature maintained in the capillary tube.
5. Having selected the desired flow rate, end conditions, and friction factor for the capillary tubing to be used, the correct length of capillary may be computed from the following equations:

$$x_3^2 + \left[\frac{2v_{f_3}}{v_{fg_3}} + \frac{2gJ\, h_{fg_3}}{G^2 (v_{fg_3})^2}\right] x_3 - \frac{2gJ\,(h_{f_2} - h_{f_3})}{G^2\,(v_{fg_3})^2} = 0$$

$$v_3 = v_{f_3} + x_3 v_{fg_3}$$

$$v_{av.} = v_1 + v_3/16$$

$$\Delta p_m = \frac{G^2}{g} (v_3 - v_1)$$

$$\Delta p_f = (p_1 - p_3) - \Delta p_m$$

$$L = \frac{gD \Delta p_f}{2G^2 f v_{av}}$$

A complete list of the symbols used in these equations may be found in the appendix. The equation for finding the average specific volume is only a rough approximation, and limits the accuracy of the calculation. This equation was checked against the integrated average specific volumes in the results with no greater than 10 per cent error.

6. Further study of the problem and the application of dimensional analysis may provide a means of accurately determining the average specific volume. The design of the capillary tube could then be made within the accuracy desired.

RECOMMENDATIONS

Equipment

1. In order to determine the exact point of flash-off in the capillary, a more accurate temperature or pressure measuring instrument must be installed. Pressure measuring devices which may solve this problem are strain gages. These could be mounted along the tube at various increments of length much the same as the thermocouples. However, they would be subject to the limitation of this spacing, and would have to be spaced very close together for a true determination of the flash-off point. A better solution to this problem, if the mechanical details can be worked out, would be to mount the capillary inside a larger bore tube. Within the annular space between the tubes, a resistance thermometer would be mounted in such a way that it could be moved along the capillary tube. This would permit the taking of temperature observations at as many points as desired, and, in particular, a "search" of the tube could be made in the vicinity of the flash-off point.
2. To improve the accuracy of the primary refrigerant calorimeter, an integrating wattmeter, or watthour meter calibrated for the blade heater resistance should be installed in the system. A variac for power input control would also be an improvement over the present rheostat control.
3. It would be desirable to have a check on the mass rate of flow other than the condenser balance. This could be accomplished by the installation of a rotameter, calibrated for use with Freon-12, in the system.

4. To eliminate the slight variations in discharge pressure in the present set-up, a throttle valve on the high pressure side of the system, with the possibility of by-pass to the low pressure side is recommended.

Future Investigation

1. The effect of increasing the pressure differential across the tube has been shown. However, due to the lack of time, investigations were made at only two discharge pressures, namely, 100 and 120 pounds gage. Further tests at discharge pressures of 80 and 140 pounds gage should permit the exact effect of this variable to be determined.
2. Restrictor tubing in 5, 4, 3, and 2 foot lengths is available in the Refrigeration and Air Conditioning Laboratories. Investigation of these tubes would determine the effect of L/D ratio, and provide a check on the analysis presented here for the 6 foot tube.
3. Finally, if the method herein presented is found to give accurate results on these tubes, a mathematical determination of the curves of specific volume vs. length for section 2-3 of the tube should be worked out. This would permit the design of an adiabatic capillary tube to the desired degree of accuracy.

APPENDIX

SUPPLEMENTARY INTRODUCTION

The ultimate object of this thesis was to determine and present a method for selecting the proper capillary tube for liquid refrigerant control in an open refrigeration system. A relationship between the bore and length of a capillary tube was available in an article entitled "Theory and Use of a Capillary Tube for Liquid Refrigerant Control" written by L. A. Staebler. The relationship is useful for obtaining the relative lengths at various bores of capillaries necessary for equivalent flow capacity. Consequently, for this study it seemed appropriate to select capillary tubes of varying lengths but with a constant diameter. Five capillary tubes with a diameter of 0.055 inches and varying in length from two to six feet were available. The six foot tube had already been installed in the experimental system.

At the outset, the intent was to begin the experimental work with the six foot tube. When sufficient data were obtained from this tube, the five foot tube was to be installed. This process of changing tubes was to continue as time and completeness of data permitted.

The experimental apparatus comprised a typical open refrigerating system with measuring instruments installed wherever specific data are needed. The individual pieces of equipment embodied in the system are as follows:

1. A Westinghouse 1 1/2 HP, compound wound, DC electric motor with stove resistance box and field rheostat.
2. A Chrysler Airtemp high-speed, four cylinder, radial compressor.
3. An oil separator.

4. A 14 tube, horizontal, concentric condenser, mounted in an insulated sheet metal box, with cooling water flow through the internal pipe and refrigerant counterflow through the annular space.
5. An electrimatic water control valve for condenser cooling water flow regulation to maintain any desired condenser refrigerant pressure.
6. A 25 pound capacity receiver.
7. A water-refrigerant heat exchanger equipped with an aquastat to control subcooling.
8. A sight glass to check liquid refrigerant conditions at capillary tube inlet.
9. The capillary tube, sealed within an insulated plywood box.
10. A primary refrigerant calorimeter with two vertical heating elements, to determine the evaporator load.
11. A wattmeter to measure the electrical power input.
12. A rheostat to control electrical power input.
13. Seven pressure gages.
14. Forty-seven copper-constantan thermocouples and a potentiometer.
15. A condenser water measuring tank and scales.

A complete description of the apparatus as well as a schematic diagram is available in an S.M. thesis by S. Klion and E. Hanley. Photographs of the existing apparatus are shown on the following pages.

The experimental apparatus permitted the variation within limits of three factors, namely the speed of the compressor, the energy

input and therefore the evaporator load, and the discharge pressure of the compressor. There was no definite starting point for the obtaining of data. The work done by Klion and Hanley consisted principally of determining the pressure-length curves for each of their runs. In addition they obtained the refrigerant flow rate at one inlet pressure but at varying calorimeter loads and compressor speeds. Their plots indicated the trend of the flash-off point. Although the system originally had a water-refrigerant heat exchanger to control the subcooling, Klion and Hanley had no success with it and made all their runs without any control over the degree of subcooling.

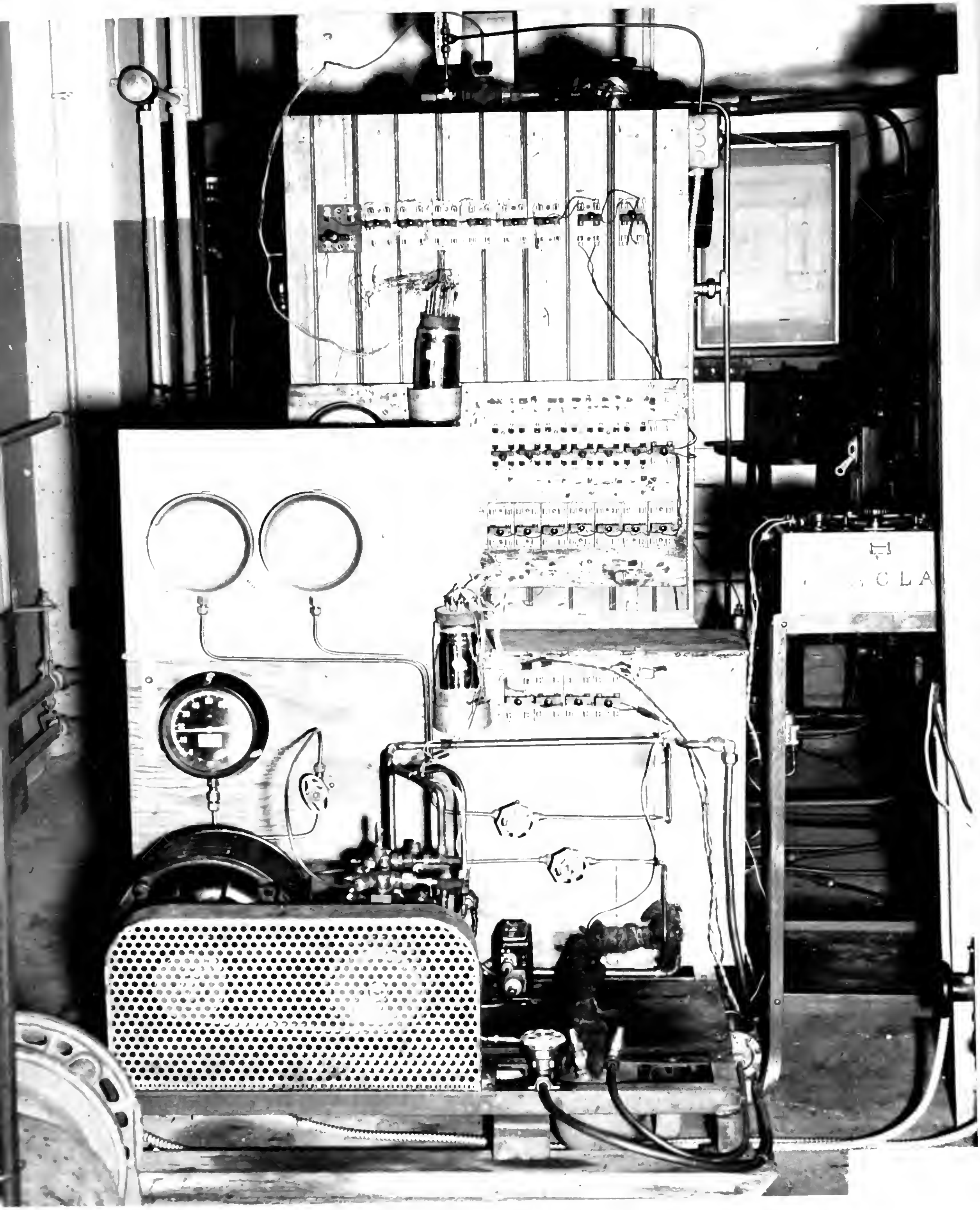
In typical refrigerating systems, an expansion valve is the device used for metering the flow of liquid refrigerant. The proper size is selected from tables prepared by the manufacturer. To select an expansion valve for any particular system, these tables are entered with the value of the desired pressure drop across the valve and the value of the maximum tonnage of the system. Their intersection in the tables indicates the proper size of expansion valve necessary for that particular system.

Accordingly, for this study it was decided to make the runs at two inlet tube pressures, at two compressor speeds, and at varying evaporator loads. From the initial runs, the correlation of data indicated that the degree of subcooling had a direct effect upon the refrigerant flow rate. Consequently, changes were made to the original heat exchanger for controlling the subcooling. The result was the ability to control the subcooling within one degree.

The many variables, dependent as well as independent, involved in the problem made it necessary to make all the runs on only one

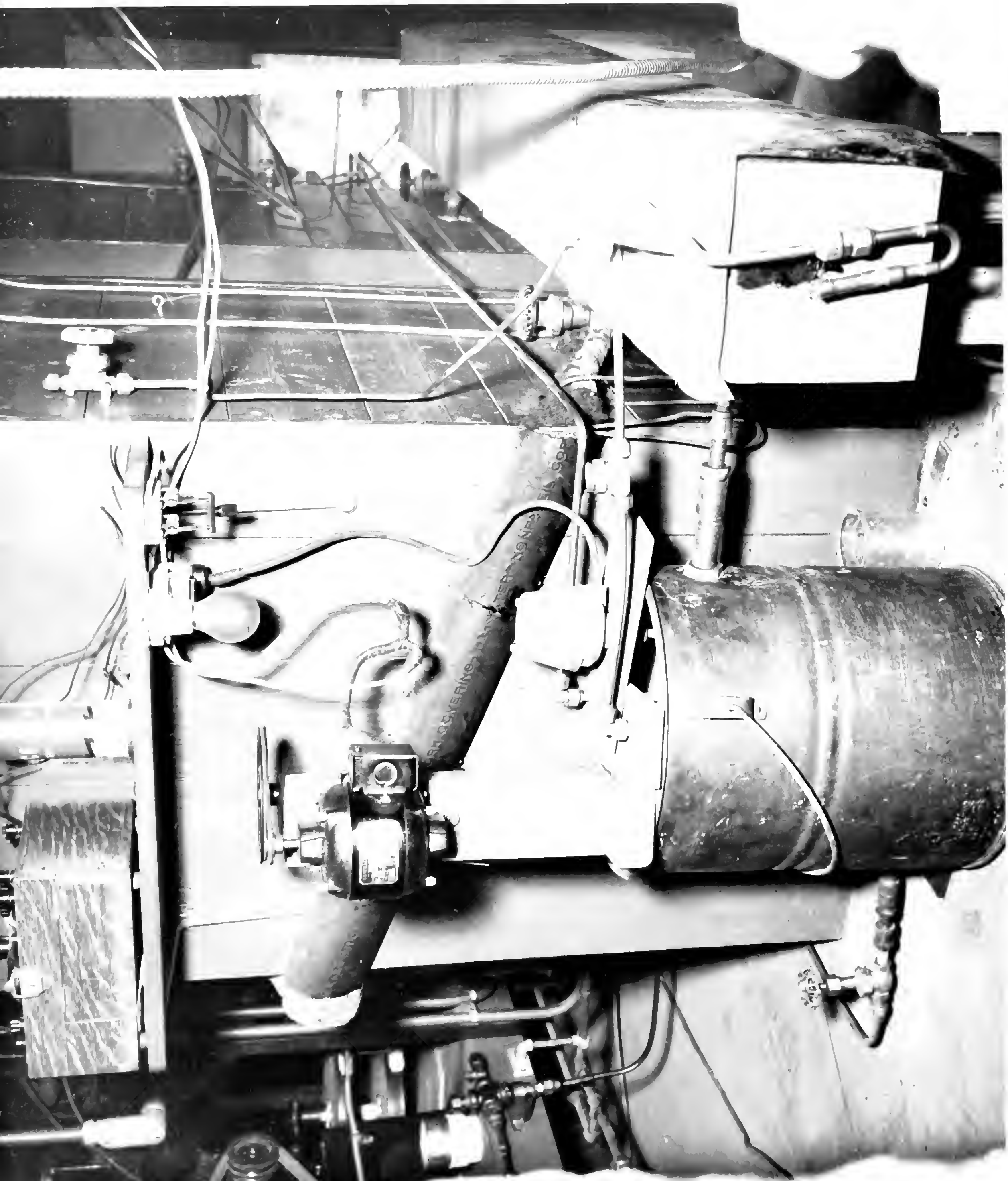
capillary tube. Correlation of the data indicated that a definite introduction had been made in the study of the flow processes in a capillary tube.

FIGURE 5



Condensing Unit, Calorimeter, and Thermocouple Banks

FIGURE 6



Heat Exchanger for Subcooling Control

FIGURE 7

Capillary Tube Installation

DETAILS OF PROCEDURE

This thesis is a continuation of an investigation of the characteristics and analysis of capillary tubes for liquid refrigerant control begun by Sanford Klion and Edward Hanley. The apparatus used in the present investigation was originally set up by Klion and Hanley. Some modifications in the original equipment were found necessary as experimentation progressed.

Early in the investigation it was felt worthwhile to install an oil separator in the refrigerant line immediately after the compressor, in order to eliminate oil in solution with the refrigerant, because the viscosity of Freon is affected by oil in solution and is one of the properties affecting an analysis of refrigerant flow in capillary tubing.

As the investigation proceeded, it was found that subcooling had a major effect on the refrigerant flow rate. This had been known from the work of other investigators, but the magnitude of the effect was not realized until several runs had been made with uncontrolled subcooling. The effect became particularly noticeable when the use of higher discharge pressures caused subcooling to become excessively large when uncontrolled. The water bath, which had been left dry, was filled with water, and a 250-watt immersion blade heater controlled by an immersion aquastat was used to hold the water bath at the proper temperature to give the desired subcooling.

A lack of conformity between the readings of the tube outlet and calorimeter inlet thermocouples, in addition to a gradual increase in the differential between those readings cast suspicion on the accuracy of thermocouple #28, and led to its replacement after run #9. Fortunately, the thermocouple was accessible without entering

the sealed box holding the tube. Replacement was made with the heavier wire used everywhere else in the system except the tube proper.

Failure of some tube thermocouples to give any reading at all, and disagreement among others which should have given the same reading led to a complete checking of the immersion of the reference junctions for all thermocouples. Some were found to be pulled completely out of their glass tubes, and many were sufficiently high to affect their readings materially.

Observation of an apparent pressure rise between the tube and the calorimeter gave rise to doubts of the accuracy of the small scale gage indicating capillary outlet pressure. This gage was therefore replaced by a large scale compound gage, on which tenths could be estimated with very good accuracy. However, the second gage seemed to indicate a similar pressure rise. Because of the press of time, it was felt that an investigation into probable velocity effects because of faulty connections at the capillary tube would not be worthwhile.

Test Procedure

All runs were made in essentially the same manner, differing mainly only in the setting of the variables. In the first runs, with compressor discharge pressure fixed around 100 psig, and with subcooling allowed to vary at random, only compressor speed and calorimeter load were controlled as variables. Later, discharge pressure was raised to 120 psig, and subcooling was made a very strictly controlled variable.

The actual procedure for obtaining data was simple but time-consuming:

1. Start up the equipment and set the variables.
2. Wait three to six hours for the system to reach equilibrium.
3. Record data in the form shown by the sample data sheets for run #15.

Evaluation of Data

Run #1 was in the nature of a practice run, and the data taken are not considered reliable. Run #7 was made with an extremely low calorimeter load; as a consequence the run had to be discontinued halfway through, to avoid liquid carryover into the compressor suction. Therefore, the average data tables do not show data for either of these two runs.

From run #1 through run #9, thermocouple #28 is known to be faulty; its readings are worthless for these runs. In all runs, variation of any thermocouple from the average before flash-off indicates poor immersion of the reference junction and a faulty reading.

For all runs, the tube exit pressure reading is questionable.

During the later runs, control of the tube entrance temperature by the water bath was excellent, but the automatic discharge pressure control was sometimes faulty, giving a varying tube entrance pressure and a consequent varying subcooling. It was found in the last runs that the longer the apparatus was in operation for any run, the more steady the discharge pressure became, and the more reliable the data for that run were.

TEST DATA - TEMPERATURES

(Degrees Fahrenheit)

		Thermocouple	Run 2	Run 3	Run 4
Capillary tube:	Inlet	1	81.7	80.0	80.1
		2	81.7	80.0	80.1
		3	81.7	80.0	80.1
		4	81.7	80.0	80.1
		5	81.7	80.0	80.1
		6	81.7	80.0	80.1
		7	81.7	80.0	80.1
		8	81.7	80.0	80.1
		9	81.7	80.0	80.1
		10	81.7	80.0	80.1
		11	81.7	80.0	80.1
		12	81.7	80.0	80.1
		13	81.7	80.0	80.1
		14	81.7	80.0	80.1
		15	81.7	80.0	80.1
		16	81.1	80.0	77.3
		17	72.4	74.9	69.9
		18	68.4	68.9	67.3
		19	65.9	65.9	64.7
		20	63.5	61.5	62.5
		21	60.9	58.9	60.4
		22	58.9	56.4	57.8
		23	55.9	54.0	55.2
		24	53.1	51.3	52.6
		25	48.7	50.2	45.4
		26	46.2	44.7	44.2
		27	39.3	36.5	38.8
	Outlet	28	(9.7)	(7.0)	(8.7)
Condenser:	Refrigerant in	31	152.4	130.5	147.3
	Water out	32	92.8	91.2	93.8
	Water in	33	62.6	60.7	56.8
	Refrigerant out	34	84.2	85.3	85.1
Compressor:	Discharge	35	174.1	144.9	167.4
	Suction	36	100.0	50.2	92.3
Calorimeter:	Top	41	103.0	13.0	90.7
		42	97.2	13.0	84.9
		43	86.8	13.0	74.4
		44	68.1	12.0	55.4
		45	42.6	12.0	31.3
		46	16.3	12.0	13.0
		47	14.2	12.0	12.2
		48	14.2	12.0	12.0
		49	14.2	12.0	12.0
		50	14.2	12.0	12.0
		51	14.2	12.0	12.0
		52	14.2	12.0	12.0
	Exit	53	117.3	16.7	105.2

TEST DATA - TEMPERATURES

(Degrees Fahrenheit)

Thermocouple	Run 5	6	8	9	10	11
1	81.8	82.3	82.3	94.1	88.4	89.4
2	81.8	82.3	82.3	94.1	88.4	89.4
3	81.8	82.3	82.3	94.1	88.4	89.4
4	81.8	82.3	82.3	94.1	88.4	89.4
5	81.8	82.3	82.3	94.1	88.4	89.4
6	81.8	82.3	82.3	94.1	88.4	89.4
7	81.8	82.3	82.3	94.1	88.4	89.4
8	81.8	82.3	82.3	94.1	88.4	89.4
9	81.8	82.3	82.3	94.1	88.4	89.4
10	81.8	82.3	82.3	94.1	88.4	89.4
11	81.8	82.3	82.3	94.1	88.4	89.4
12	81.8	82.3	82.3	94.1	88.4	89.4
13	81.8	82.3	82.3	92.7	88.4	89.4
14	81.8	82.3	82.3	86.0	88.4	89.4
15	81.8	82.3	82.3	83.2	88.4	89.4
16	81.8	78.0	77.1	81.8	88.4	88.5
17	73.3	71.4	71.4	79.0	81.8	82.6
18	-	68.6	-	77.1	78.5	78.3
19	66.7	65.6	66.2	74.7	75.2	75.7
20	63.7	63.1	63.2	71.2	72.8	73.2
21	59.9	60.7	61.2	69.1	71.0	71.4
22	57.2	58.7	58.2	66.7	69.0	69.4
23	56.2	56.2	56.3	64.6	66.8	67.2
24	52.8	52.3	52.8	61.2	64.1	64.4
25	49.6	58.8	58.8	56.8	60.7	60.7
26	45.4	45.4	42.9	51.9	56.7	56.6
27	40.4	39.4	39.0	47.1	50.8	50.0
28	(7.1)	(3.2)	(-7.0)	(0.3)	(1.2)	20.8
31	147.8	138.9	124.9	140.8	151.1	142.2
32	95.1	94.1	92.0	107.0	107.8	106.6
33	53.4	54.5	54.5	58.8	57.8	57.0
34	86.8	87.1	87.1	99.8	98.6	99.3
35	189.0	173.2	150.4	170.9	184.1	170.0
36	106.9	65.9	50.2	41.2	79.0	47.6
41	121.8	38.2	11.6	18.3	65.0	20.3
42	116.6	32.9	11.6	17.3	59.5	19.9
43	107.8	27.0	11.6	17.7	51.0	20.0
44	87.8	14.2	10.7	17.7	33.5	19.4
45	60.1	10.2	10.7	17.7	21.2	19.4
46	26.2	10.2	11.6	17.7	20.9	19.4
47	12.2	10.2	11.1	17.7	20.9	19.4
48	12.5	10.2	11.1	17.3	20.9	19.8
49	12.5	11.6	11.6	17.7	20.9	20.0
50	12.5	10.2	11.1	17.7	20.9	20.0
51	12.5	10.2	11.1	17.7	20.9	20.0
52	12.5	11.6	11.6	17.7	20.9	20.3
53	134.8	50.4	18.3	22.0	74.9	20.2

TEST DATA - TEMPERATURES
(Degrees Fahrenheit)

Thermocouple	Run 12	Run 13	Run 14	Run 15
1	97.0	90.3	95.6	95.2
2	97.0	90.3	95.6	95.2
3	97.0	90.3	95.6	95.2
4	97.0	90.3	95.6	95.2
5	97.0	90.3	95.6	95.2
6	97.0	90.3	95.6	95.2
7	97.0	90.3	95.6	95.2
8	97.0	90.3	95.6	95.2
9	97.0	90.3	95.6	93.2
10	97.0	90.3	95.6	90.4
11	97.0	90.3	93.9	88.5
12	92.3	90.3	88.6	86.6
13	87.5	90.3	86.6	85.1
14	85.8	90.3	-	83.7
15	84.2	90.3	83.8	82.3
16	82.3	89.4	81.7	80.9
17	80.9	84.2	79.7	79.0
18	78.5	79.5	77.4	76.2
19	75.2	76.6	74.1	73.8
20	71.9	73.8	71.5	70.0
21	69.6	72.4	69.1	68.1
22	67.7	70.0	66.8	66.0
23	64.6	68.1	64.4	63.2
24	62.0	65.6	61.6	60.7
25	58.2	62.7	57.6	56.8
26	54.3	58.2	54.1	51.9
27	47.3	50.8	48.8	45.9
28	18.7	25.0	24.6	19.0
31	150.7	137.9	155.4	160.0
32	109.8	107.3	110.5	111.0
33	56.2	54.4	58.8	59.2
34	100.1	100.6	100.6	100.6
35	188.4	163.0	194.3	205.0
36	92.4	48.0	102.6	108.1
41	90.5	24.1	108.0	121.0
42	82.6	22.5	102.0	115.8
43	73.4	23.3	92.6	106.2
44	55.5	23.3	72.7	86.2
45	33.5	22.9	49.3	61.3
46	19.1	22.9	26.2	29.4
47	18.2	22.9	24.0	18.6
48	18.2	23.3	24.0	18.6
49	18.2	23.3	24.0	18.6
50	18.2	23.3	24.0	18.6
51	18.2	23.3	24.0	18.6
52	18.2	23.3	24.0	18.6
53	103.1	26.8	122.0	133.4

TEST DATA - TEMPERATURES

(Degrees Fahrenheit)

Thermocouple	Run 16	Run 17	Run 18	Run 19
1	95.0	81.3	81.8	80.9
2	95.0	81.3	81.8	80.9
3	95.0	81.3	81.8	80.9
4	95.0	81.3	81.8	80.9
5	95.0	81.3	81.8	80.9
6	95.0	81.3	81.8	80.9
7	95.0	81.3	81.8	80.9
8	95.0	81.3	81.8	80.9
9	95.0	81.3	81.8	80.9
10	95.0	81.3	81.8	80.9
11	95.0	81.3	81.8	80.9
12	88.4	81.3	81.8	80.9
13	86.2	81.3	81.8	80.9
14	84.9	81.3	80.9	80.9
15	83.0	81.3	78.6	80.9
16	81.6	81.3	73.4	80.9
17	79.8	80.5	71.1	80.1
18	77.6	72.2	68.9	73.2
19	74.5	67.5	66.4	67.9
20	71.2	65.6	63.0	64.9
21	69.0	62.8	61.3	63.0
22	67.5	61.2	59.2	61.6
23	64.5	58.9	56.8	59.1
24	61.6	56.0	53.9	56.7
25	58.1	52.9	50.6	52.8
26	53.7	48.9	47.0	59.3
27	47.2	42.5	40.2	43.3
28	17.0	18.2	16.2	19.4
31	146.2	148.4	139.4	149.8
32	109.0	95.1	93.3	95.3
33	57.2	60.0	63.4	60.0
34	100.0	88.1	88.1	88.4
35	179.2	190.3	176.4	190.5
36	54.4	68.1	59.8	94.3
41	18.6	43.6	25.3	90.9
42	16.5	38.4	21.9	84.7
43	16.6	30.5	18.4	73.7
44	15.7	18.7	14.7	52.1
45	15.7	17.5	15.1	33.6
46	15.9	17.5	14.7	19.2
47	15.9	17.5	14.5	18.7
48	16.3	17.5	15.4	18.8
49	16.7	17.5	16.2	18.8
50	16.3	17.5	15.0	18.8
51	16.7	17.5	15.5	18.8
52	17.0	17.5	15.4	18.8
53	22.2	55.3	36.1	107.0

TEST DATA

Run		2	3	4
Barometer	" Hg	30.21	30.32	30.19
Room temperature	°F	79.0	75.3	77.0
Water rate	lb/hr	109.6	97.1	87.1
Calorimeter input	watts	895.	698.	910.
Compressor speed	rpm	1230.	1190.	1243.
Absolute pressures:	psia			
Calorimeter exit		28.3	28.2	27.8
Compressor suction		28.0	28.1	27.4
Condenser inlet		114.3	113.7	113.1
Condenser outlet		113.8	112.9	112.0
Tube inlet		110.8	110.7	110.4
Tube outlet		27.4	27.6	26.9

Run		5	6	8
Barometer	" Hg	30.45	30.44	30.05
Room temperature	°F	75.0	76.0	75.5
Water rate	lb/hr	74.3	70.6	77.0
Calorimeter input	watts	991.	764.	709.
Compressor speed	rpm	1215.	1216.	1207.
Absolute pressures:	psia			
Calorimeter exit		29.3	28.6	28.7
Compressor suction		29.1	28.3	28.2
Condenser inlet		114.6	115.1	115.3
Condenser outlet		114.0	114.4	114.4
Tube inlet		111.3	111.8	112.0
Tube outlet		28.5	26.5	28.1

Run		9	10	11
Barometer	" Hg	29.61	29.54	30.27
Room temperature	°F	78.0	76.2	76.0
Water rate	lb/hr	58.3	66.9	64.0
Calorimeter input	watts	704.	979.	798.
Compressor speed	rpm	1182.	1213.	1210.
Absolute pressures:	psia			
Calorimeter exit		32.7	35.0	33.9
Compressor suction		32.1	34.7	33.2
Condenser inlet		136.4	135.5	135.2
Condenser outlet		135.4	135.2	134.4
Tube inlet		133.3	132.9	132.5
Tube outlet		32.6	34.1	33.1

TEST DATA

Run		12	13	14
Barometer	" Hg	29.47	29.88	29.78
Room temperature	°F	75.0	76.9	77.0
Water rate	lb/hr	54.1	62.0	58.6
Calorimeter input	watts	873.	873.	979.
Compressor speed	rpm	1207.	1206.	1211.
Absolute pressures:	psia			
Calorimeter exit		33.3	37.0	37.2
Compressor suction		32.7	36.1	37.0
Condenser inlet		136.6	137.1	137.4
Condenser outlet		136.0	136.3	137.1
Tube inlet		134.3	134.6	135.2
Tube outlet		32.4	36.2	36.2

Run		15	16	17
Barometer	" Hg	30.30	30.31	29.58
Room temperature	°F	77.2	76.0	80.8
Water rate	lb/hr	58.3	56.1	93.0
Calorimeter input	watts	980.	703.	835.
Compressor speed	rpm	1407.	1400.	1401.
Absolute pressures:	psia			
Calorimeter exit		32.6	31.5	33.1
Compressor suction		32.4	31.4	33.1
Condenser inlet		137.9	137.2	116.1
Condenser outlet		137.2	137.1	115.3
Tube inlet		135.4	134.7	112.9
Tube outlet		31.4	31.6	32.1

Run		18	19
Barometer	" Hg	29.94	29.80
Room temperature	°F	81.5	76.5
Water rate	lb/hr	101.9	95.3
Calorimeter input	watts	740.	990.
Compressor speed	rpm	1423.	1420.
Absolute pressures:	psia		
Calorimeter exit		31.1	33.7
Compressor suction		31.4	33.9
Condenser inlet		115.7	116.9
Condenser outlet		114.9	115.9
Tube inlet		112.3	113.4
Tube outlet		30.4	32.9

SAMPLE DATA SHEET

RUN #15 Date: March 19, 1949

Barometer: 30.30" Hg Tube Length: 6'

Room temperature: 77.2 °F Tube diameter: 0.055"

Time	0	15	30	45	60	75	Average	
Thermocouple	mv	mv	mv	mv	mv	mv	mv	°F
Capillary tube								
1 Inlet	1.29	1.31	1.31	1.30	1.30	1.31	1.30	95.2
2	1.29	1.31	1.31	1.30	1.30	1.31	1.30	95.2
3	1.29	1.31	1.31	1.30	1.30	1.31	1.30	95.2
4	1.30	1.30	1.31	1.30	1.30	1.31	1.30	95.2
5	1.30	1.30	1.31	1.30	1.30	1.31	1.30	95.2
6	1.30	1.30	1.31	1.30	1.30	1.31	1.30	95.2
7	1.30	1.30	1.31	1.30	1.30	1.31	1.30	95.2
8	1.29	1.30	1.31	1.30	1.30	1.31	1.30	95.2
9	1.27	1.27	1.25	1.26	1.27	1.27	1.26	93.2
10	1.20	1.19	1.20	1.20	1.21	1.20	1.20	90.4
11	1.17	1.17	1.16	1.15	1.17	1.17	1.16	88.5
12	1.13	1.11	1.13	1.11	1.10	1.12	1.12	86.6
13	1.10	1.09	1.10	1.09	1.08	1.10	1.09	85.1
14	1.08	1.08	1.07	1.05	1.04	1.06	1.06	83.7
15	1.03	1.03	1.03	1.03	1.02	1.02	1.03	82.3
16	0.99	1.01	0.99	1.00	0.99	0.99	1.00	80.9
17	0.95	0.95	0.95	0.97	0.97	0.96	0.96	79.0
18	0.90	0.90	0.89	0.90	0.92	0.91	0.90	76.2
19	0.85	0.85	0.84	0.85	0.86	0.85	0.85	73.8
20	0.76	0.76	0.78	0.79	0.78	0.76	0.77	70.0
21	0.72	0.72	0.73	0.74	0.74	0.73	0.73	68.1
22	0.69	0.69	0.67	0.69	0.70	0.68	0.69	66.0
23	0.63	0.64	0.62	0.63	0.65	0.63	0.63	63.2
24	0.57	0.59	0.58	0.58	0.59	0.57	0.58	60.7
25	0.49	0.49	0.49	0.50	0.51	0.50	0.50	56.8
26	0.40	0.43	0.40	0.40	0.40	0.40	0.40	51.9
27	0.28	0.28	0.26	0.29	0.29	0.27	0.28	45.9
28 Outlet	-0.30	-0.30	-0.31	-0.31	-0.30	-0.31	-0.30	19.0
Condenser								
31 Ref. in	3.14	3.15	3.15	3.16	3.17	3.17	3.15	160.0
32 Water out	1.90	1.90	1.89	1.89	1.88	1.88	1.89	111.0
33 Water in	0.64	0.63	0.63	0.63	0.63	0.63	0.63	59.2
34 Ref. out	1.62	1.63	1.63	1.63	1.63	1.63	1.63	100.6
Compressor								
35 Discharge	4.30	4.33	4.35	4.35	4.37	4.37	4.35	205.0
36 Suction	1.81	1.82	1.80	1.83	1.84	1.82	1.82	108.1

SAMPLE DATA SHEET

(Continued)

Time	0	15	30	45	60	75	Average	
Thermocouple	mv	mv	mv	mv	mv	mv	mv	°F
Calorimeter								
41 Top	2.13	2.12	2.11	2.14	2.17	2.15	2.14	121.0
42	2.00	2.00	1.99	2.03	2.02	2.02	2.01	115.8
43	1.76	1.76	1.75	1.79	1.79	1.78	1.77	106.2
44	1.26	1.26	1.26	1.30	1.30	1.28	1.28	86.2
45	0.66	0.64	0.68	0.72	0.68	0.67	0.68	61.3
46	-0.05	-0.09	-0.05	-0.05	-0.06	-0.06	-0.06	29.4
47	-0.30	-0.30	-0.32	-0.32	-0.31	-0.33	-0.31	18.6
48	-0.32	-0.31	-0.32	-0.33	-0.31	-0.33	-0.31	18.6
49	-0.32	-0.31	-0.32	-0.33	-0.31	-0.33	-0.31	18.6
50	-0.32	-0.31	-0.32	-0.33	-0.31	-0.33	-0.31	18.6
51	-0.32	-0.31	-0.32	-0.33	-0.31	-0.33	-0.31	18.6
52 Bottom	-0.32	-0.31	-0.32	-0.33	-0.31	-0.33	-0.31	18.6
53 Exit	9.85	9.69	9.87	9.96	9.84	9.84	4)9.84	
							2.46	133.4

Time	Water	Difference	Water Rate	Power	Speed
minutes	lbs	lbs	lb/min	watts	rpm
0	0.9			1010	1404
5	5.9	5.0	1.00	1020	1400
10	10.8	4.9	0.98	995	1398
15	15.6	4.8	0.96	1000	1410
20	20.3	4.7	0.94	1005	1410
25	25.1	4.8	0.96	1010	1402
30	30.0	4.9	0.98	1010	1400
35	34.9	4.9	0.98	1005	1410
40	39.8	4.9	0.98	1005	1400
45	44.6	4.8	0.96	1010	1400
50	49.5	4.9	0.98	1010	1410
55	54.5	5.0	1.00	1005	1408
60	59.0	4.5	0.90	1010	1412
65	63.9	4.9	0.98	1010	1412
70	68.6	4.7	0.94	1015	1410
75	73.4	4.8	0.96	1020	1410
80	78.4	5.0	1.00	1000	1410
85	83.2	4.8	0.96	1000	1410
90	87.9	4.7	0.94	1000	1412
Difference	87.0				
Average				1007	
Correction	+0.5			-27	
Average	87.5		0.972	980	1407

SAMPLE DATA SHEET

(Concluded)

RUN #15

Time	Calorimeter Exit	Compressor Suction	Condenser Inlet	Condenser Outlet	Tube Inlet	Tube Outlet
minutes	psig	psig	psig	psig	psig	psig
0	17.3	17.2	122.3	121.9	121.2	16.7
5	17.4	17.3	122.2	121.8	121.1	16.7
10	17.4	17.2	122.2	121.8	121.1	16.4
15	17.3	17.1	122.4	122.0	121.3	16.3
20	17.3	17.1	122.4	122.0	121.3	16.4
25	17.3	17.2	122.3	121.9	121.2	16.7
30	17.4	17.3	122.6	122.1	121.3	16.8
35	17.6	17.4	122.7	122.2	121.3	16.9
40	17.7	17.5	122.3	121.8	121.1	16.8
45	17.4	17.2	122.2	121.6	121.1	16.5
50	17.2	16.9	122.2	121.6	121.0	16.2
55	17.0	16.9	122.3	121.8	121.1	16.2
60	17.1	16.9	122.7	122.1	121.3	16.3
65	17.2	17.1	122.4	121.9	121.2	16.4
70	17.3	17.2	122.8	122.0	121.4	16.6
75	17.4	17.3	122.3	121.8	121.1	16.6
80	17.2	17.1	122.2	121.6	121.0	16.4
85	17.1	16.9	122.1	121.3	120.9	16.2
90	16.8	16.8	122.2	121.6	121.0	16.1
Average	17.3	17.1	122.4	121.8	121.2	16.5
Correction	+0.45	+0.44	+0.60	+0.53	-0.70	0.00
Gage	17.75	17.54	123.00	122.33	120.50	16.50
Barometer	14.88	14.88	14.88	14.88	14.88	14.88
Absolute	32.63	32.42	137.88	137.21	135.38	31.38

SAMPLE CALCULATIONS
(Test Run #15)

	Point	Temperature $t \sim {}^\circ F$	Pressure $p \sim psia$	Enthalpy $h \sim BTU/lb$	Specific Volume $v \sim ft^3/lb$
A.	Calorimeter Exit	133.4	32.6	97.45	1.576
B.	Compressor Suction	108.1	32.4	93.57	1.510
C.	Condenser Inlet	160.0	137.9	98.40	0.356
D.	Condenser Outlet	100.6	137.2	31.31	0.0127
1.	Tube Inlet	95.2	135.4	29.98	0.0126
2.	Tube Flash-off	95.2	123.1	29.98	0.0126
3.	Tube Outlet	34.0	46.4	—	—
E.	Calorimeter Inlet	18.6	32.6	29.98	—

1. CALORIMETER LOAD

$$Q_{cal} = 3413\,(\text{watts input}) + UA\,(t_{room} - t_{cal.\,shell})$$

$$= 3413\,(980) + 1.315\,(77.2 - 52.6)$$

$$= 3344.7 + 32.3 = 3377.0 \text{ BTU/hr}$$

2. REFRIGERANT MASS RATE OF FLOW

$$W_r = \frac{Q_{cal}}{h_A - h_E} = \frac{3377}{97.45 - 29.98} = \frac{3377}{67.47}$$

$$= 50.05 \text{ lb/hr} = 0.834 \text{ lb/min}$$

$$G = \frac{W_r}{60A} = \frac{4W_r}{60\pi\left(\frac{0.055}{12}\right)^2} = 1010\,W_r$$

$$= 1010\,(0.834) = 842.4 \text{ lb/ft}^2\text{-sec}$$

3. FRICTION FACTOR FOR SECTION 1-2 OF THE CAPILLARY

$$f_{1-2} = \frac{gD(p_1 - p_2)}{2L_{1-2}\, G^2\, v_1}$$

$$= \frac{32.2\,(0.055)(135.4 - 123.1)\,144}{24\,(3.00)(842.4)^2\,(0.0126)}$$

$$= 0.00487$$

4. REYNOLDS NUMBER FOR SECTION 1-2

$$Re_{1-2} = \frac{GD}{\mu} = \frac{842.4\,(0.055)}{(1.64 \times 10^{-4})\,12} = 2.35 \times 10^4$$

TO FIND THE FRICTION FACTOR FOR SECTION 2-3 OF THE CAPILLARY, THE TABLE ON THE FOLLOWING PAGE IS MADE UP. THE STATIONS 2, a, b, c, --- 3 ARE TAKEN AT VARIOUS INCREMENTS OF LENGTH ALONG THE TUBE FROM THE FLASH-OFF POINT AT 2 TO THE END OF THE TUBE AT 3. THE TEMPERATURES AT THESE POINTS WERE TAKEN BY MEANS OF THE THERMOCOUPLES MOUNTED ALONG THE TUBE, EXCEPT AT 3, WHERE THE TEMPERATURE WAS ESTIMATED BY DRAWING A SMOOTH CURVE THROUGH THE THERMOCOUPLE TEMPERATURES OVER THE LAST PART OF THE TUBE. THIS METHOD OF ESTIMATING THE TUBE EXIT TEMPERATURE IS ILLUSTRATED BY THE GRAPH ON THE SECOND PAGE FOLLOWING.

TABLE II

STATION	L inches	t °F	v_f ft³/lb	v_{fg} ft³/lb	h_f BTU/lb	h_{fg} BTU/lb	x quality	v ft³/lb
2	0	95.2	0.01258	0.329	29.98	58.23	0.0000	0.0126
a	6	90.4	0.01248	0.353	28.80	58.98	0.0200	0.0185
b	12	86.6	0.01241	0.373	27.87	59.56	0.0353	0.0256
c	16	83.7	0.01236	0.390	27.17	59.98	0.0467	0.0306
d	20	80.9	0.01232	0.407	26.50	60.39	0.0573	0.0355
e	24	76.2	0.01224	0.437	25.37	61.07	0.0751	0.0451
f	28	70.0	0.01210	0.481	23.90	61.92	0.0974	0.0590
g	30	66.0	0.01202	0.512	22.95	62.47	0.1115	0.0691
h	32	60.7	0.01194	0.557	21.73	63.36	0.1287	0.0836
i	34	51.9	0.01180	0.641	19.70	64.28	0.1572	0.1126
3	36	34.0	0.01152	0.865	15.65	66.40	0.2080	0.1913

THE QUALITY AND SPECIFIC VOLUME TABULATED ABOVE ARE DERIVED BY INDIVIDUAL CALCULATION FOR EACH STATION. A SAMPLE CALCULATION FOR STATION 3 IS SHOWN IN STEP 5.

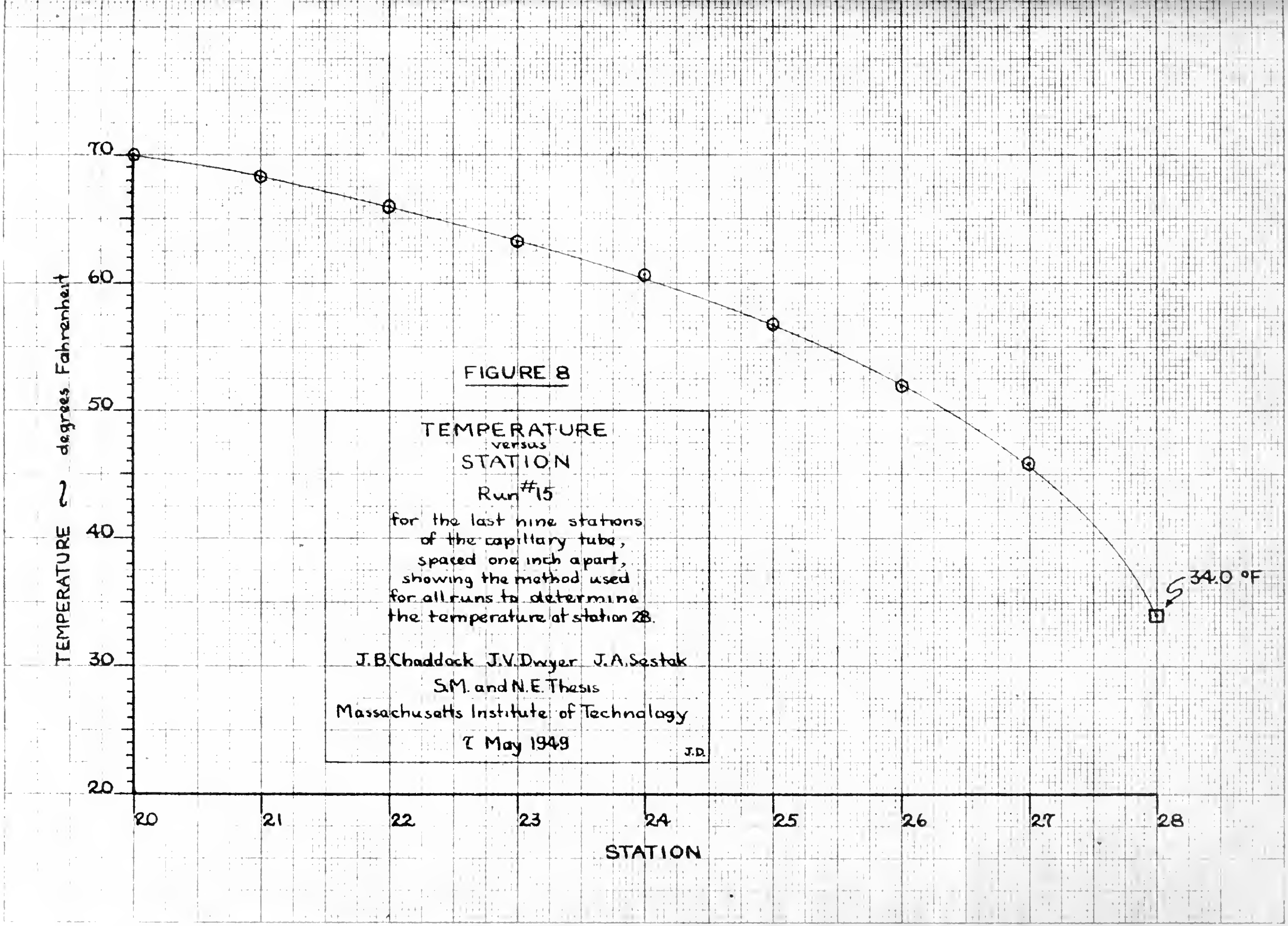


FIGURE 8
TEMPERATURE versus STATION
Run #15
for the last nine stations of the capillary tube, spaced one inch apart, showing the method used for all runs to determine the temperature at station 28.
J.B. Chaddock J.V. Dwyer J.A. Sestak
S.M. and N.E. Thesis
Massachusetts Institute of Technology
7 May 1949
J.D.
34.0 °F
TEMPERATURE ~ degrees Fahrenheit
70
60
50
40
30
20
20
21
22
23
24
25
26
27
28
STATION

5. SAMPLE CALCULATION OF QUALITY AND SPECIFIC VOLUME

$$\frac{2gJ}{G^2} = \frac{2(32.2)(778)}{(842.4)^2} = 0.0704$$

$$x_3^2 + \left[\frac{2v_{f_2}}{v_{fg_3}} + \left(\frac{2gJ}{G^2}\right)\frac{(h_{fg_3})}{(v_{fg_3})^2}\right] x_3 - \left(\frac{2gJ}{G^2}\right)\frac{(h_{f_2} - h_{f_3})}{(v_{fg_3})^2} = 0$$

$$x_3^2 + \left[\frac{2(0.01152)}{0.865} + 0.0704\frac{66.40}{(0.865)^2}\right] x_3 - 0.0704\frac{(29.98 - 15.65)}{(0.865)^2} = 0$$

$$x_3^2 + [0.0267 + 6.2488]\, x_3 - 1.3486 = 0$$

$$x_3 = \tfrac{1}{2}\left[-6.2755 + \sqrt{(6.2755)^2 + 4(1.3486)}\right]$$

$$x_3 = \tfrac{1}{2}[-6.2755 + 6.6915] = 0.2080$$

$$v_3 = v_{f_3} + v_{fg_3} = 0.01152 + 0.208\,(0.865)$$

$$= 0.01152 + 0.1798 = 0.1913 \ \text{ft}^3/\text{lb}$$

6. FRICTION FACTOR FOR SECTION 2-3 OF THE CAPILLARY

THE SPECIFIC VOLUMES AS FOUND IN TABLE II ARE PLOTTED AGAINST LENGTH FROM THE FLASH-OFF POINT, AND THE AREA UNDER THE CURVE IS OBTAINED BY MEANS OF A PLANIMETER, GIVING THE INTEGRAL $\int_2^3 v\, dL$. THE PLOT FOR THIS RUN IS SHOWN ON THE FOLLOWING PAGE.

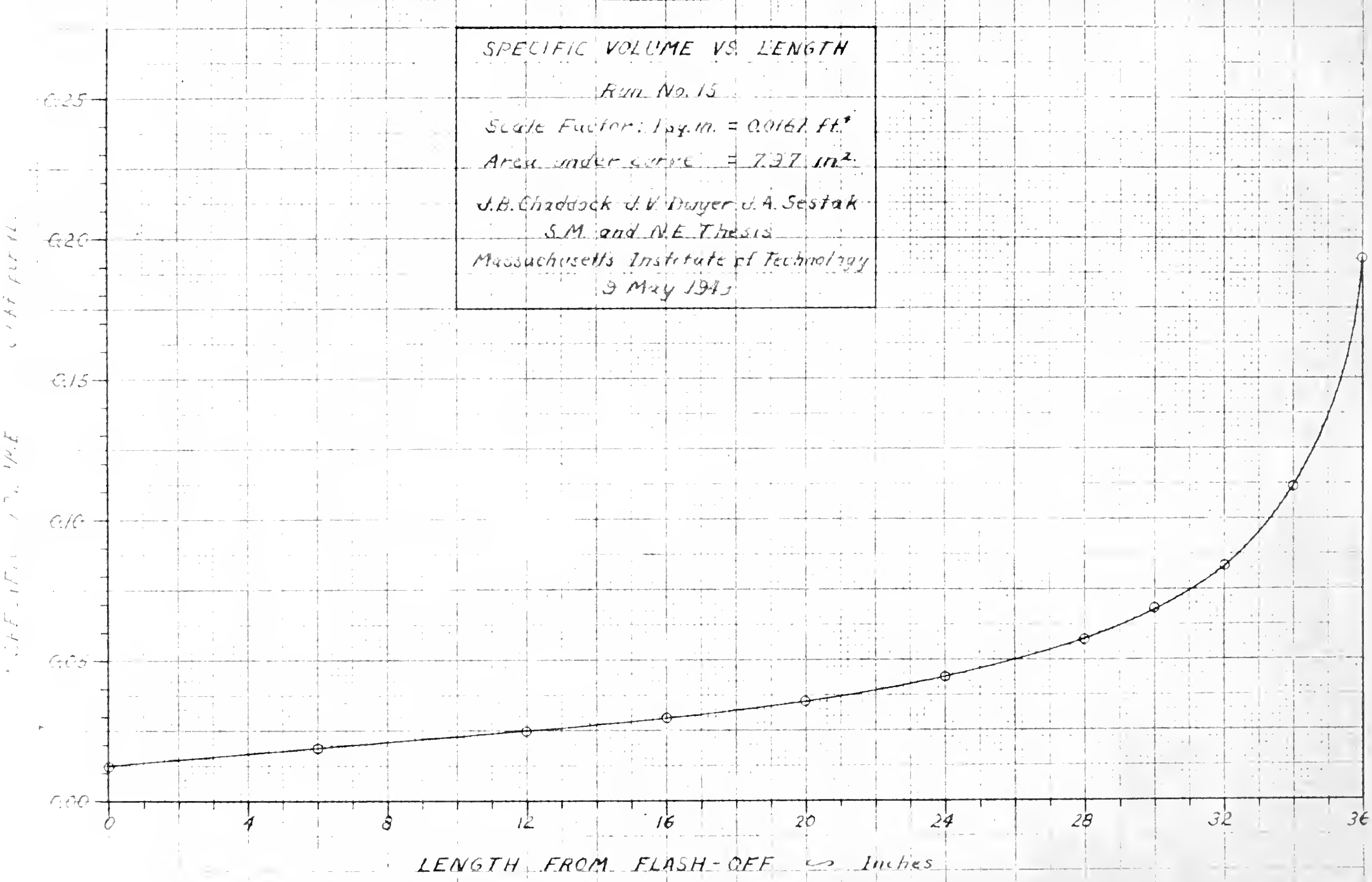
$$\frac{f_{2\text{-}3}}{r}\int_2^3 v\, dL = \frac{g}{G^2}(p_2 - p_3) - (v_3 - v_2)$$

$$f_{2\text{-}3}\left(\frac{12\times 2}{0.053}\right)[(7.97)(0.01667)]$$
$$= \frac{32.2}{(842.4)^2}(144)(123.1 - 46.4) - (0.1913 - 0.0126)$$

$$f_{2\text{-}3}\,(57.95) = 0.5007 - 0.1787$$

$$f_{2\text{-}3} = \frac{0.3220}{57.95} = 0.00555$$

FIGURE 9


SPECIFIC VOLUME VS LENGTH
Run No. 15
Scale Factor: 1 sq. in. = 0.0167 ft.³
Area under curve = 7.37 in².
J.B. Chaddock J.V. Dwyer J.A. Sestak
S.M. and N.E. Thesis
Massachusetts Institute of Technology
9 May 1943
0.25
0.20
0.15
0.10
0.05
0.00
0
4
8
12
16
20
24
28
32
36
LENGTH FROM FLASH-OFF ~ Inches

7. AVERAGE FRICTION FACTOR FOR TUBE

APPLYING THE MOMENTUM EQUATION WITHOUT THE FRICTION FACTOR TERM, CALCULATE THE PRESSURE DROP WHICH IS DUE TO MOMENTUM CHANGE.

$$\Delta p_{mom.} = \frac{G^2}{g}(v_3 - v_2)$$

$$= \frac{(842.4)^2}{144\ (32.2)}(0.1913 - 0.0126) = 27.4 \text{ psia}$$

$$\Delta p_{frict.} = (p_1 - p_3) - \Delta p_{mom.}$$

$$= (135.4 - 46.4) - 27.4 = 61.6 \text{ psia}$$

$$v_{av.} = \frac{L_{1-2}}{L}(v_1) + \frac{L_{2-3}}{L}\left[\left(\int_2^3 v\, dL\right)\left(\frac{1}{L_{2-3}}\right)\right]$$

$$= \frac{3}{6}(0.01258) + \frac{3}{6}\left[(7.97 \times 0.01667)\left(\tfrac{1}{3}\right)\right]$$

$$= 0.00610 + 0.02234 = 0.02844 \text{ ft}^3/\text{lb}$$

$$f_{av.} = \frac{g D\,(\Delta p_f)}{2 L G^2\, v_{av.}}$$

$$= \frac{32.2\ (0.055)(61.6)\ 144}{6.00\,(24)(842.4)^2(0.02844)}$$

$$= 0.00540$$

TO ILLUSTRATE THE USE OF THE EQUATIONS FOR CALCULATING THE CORRECT LENGTH OF CAPILLARY TUBING, THE FOLLOWING CONDITIONS ARE TAKEN FROM TEST RUN # 13.

t_1 = 90.6 °F	t_3 = 30.6 °F	
p_1 = 134.6 psia	p_3 = 48.1 psia	
v_1 = 0.01248 ft³/lb	v_{f_3} = 0.01156 ft³/lb	v_{fg_3} = 0.836 ft³/lb
h_1 = 28.77 BTU/lb	h_{f_3} = 16.10 BTU/lb	h_{fg_3} = 66.17 BTU/lb
W_r = 0.966 lb/min	G = 975.4 lb/ft²-sec	t_s = 11.30 °F

8. CALCULATION OF QUALITY AND VOLUME AT TUBE EXIT

$$\frac{2gJ}{G^2} = \frac{2(32.2)(778.26)}{(975.4)^2} = 0.05253$$

$$x_3^2 + \left[\frac{2v_{f_3}}{v_{fg_3}} + \left(\frac{2gJ}{G^2}\right)\frac{(h_{fg_3})}{(v_{fg_3})^2}\right]x_3 - \left(\frac{2gJ}{G^2}\right)\frac{(h_{f_1} - h_{f_3})}{(v_{fg_3})^2} = 0$$

$$x_3^2 + \left[\frac{2(0.01156)}{0.836} + 0.05253\frac{66.17}{(0.836)^2}\right]x_3 - 0.05253\frac{(28.77 - 16.10)}{(0.836)^2} = 0$$

$$x_3^2 + [0.0276 + 4.9734]\,x_3 - 0.9523 = 0$$

$$x_3 = \tfrac{1}{2}\left[-5.0010 + \sqrt{(5.0010)^2 + 4(0.9523)}\right]$$

$$x_3 = \tfrac{1}{2}[-5.0010 + 5.3684] = 0.1837$$

$$v_3 = v_{f_3} + x_3 v_{fg_3} = 0.0116 + 0.1837(0.836)$$

$$= 0.0116 + 0.1534 = 0.1650 \text{ ft}^3/\text{lb}$$

9. PRESSURE DROP DUE TO MOMENTUM CHANGE

$$\Delta p_{mom.} = \frac{G^2}{g}(v_3 - v_1)$$

$$= \frac{(975.4)^2}{144\,(32.2)}(0.1650 - 0.0125) = 31.2 \text{ psia}$$

10. PRESSURE DROP DUE TO FRICTION

$$\Delta p_{frict.} = (p_1 - p_3) - \Delta p_{mom.}$$

$$= (134.6 - 48.1) - 31.2 = 55.3 \text{ psia}$$

WITH 11.3 °F OF SUBCOOLING AND WITH A HEAD PRESSURE OF 135 psia, THE FLASH-OFF IN AN ADIABATIC TUBE WILL OCCUR AROUND 3/4 OF THE LENGTH FROM THE ENTRANCE. THE SPECIFIC VOLUME UP TO THE FLASH-OFF POINT IS CONSTANT AT V_1. THE AVERAGE SPECIFIC VOLUME AFTER FLASH-OFF IS APPROXIMATED BY $\frac{1}{2}(V_1 + \frac{1}{2}V_3)$.

11. AVERAGE SPECIFIC VOLUME THROUGH THE CAPILLARY

$$V_{AV.} = \frac{3}{4}V_1 + \frac{1}{4}\left[\frac{1}{2}(V_1 + \frac{1}{2}V_3)\right]$$

$$= \frac{3}{4}(0.0125) + \frac{1}{4}\left[\frac{1}{2}(0.0125) + \frac{1}{4}(0.1650)\right]$$

$$= 0.00938 + 0.01125 = 0.02063 \quad ft^3/lb$$

NOW APPLY THE FRICTION FACTOR EQUATION TO COMPUTE THE LENGTH OF CAPILLARY, USING THE AVERAGE VALUE OF 0.005 FOR THE FRICTION FACTOR.

12. LENGTH OF CAPILLARY

$$L = \frac{g\,D\,\Delta p_f}{2 f_{av} G^2 V_{av}}$$

$$= \frac{(32.2)(0.055)(55.3)(144)}{24(0.005)(975.4)^2(0.02063)}$$

$$= 5.97 \text{ ft.}$$

$$\text{ERROR} = \frac{6.00 - 5.97}{6.00}(100) = 0.5\,\%$$

DISCUSSION

The adiabatic flow process in the capillary tube is more easily treated if we consider it to be made up of two processes. The first is the isothermal turbulent flow from tube entrance, point 1, to flash-off, point 2. In this section of the tube the flow follows the familiar Fanning equation:

$$\frac{-dp}{dL} = \frac{4f\,V^2}{2g\,D\,v} \tag{1}$$

For isothermal flow the volume is constant. Integrating between the end points 1 and 2 and solving for the friction factor gives:

$$f_{1-2} = \frac{2g\,D\,v_1\,(p_1 - p_2)}{4L_{1-2}\,V_1^{\,2}} \tag{2}$$

From the well known continuity equation:

$$V = \frac{w_r\,v}{A} = G\,v \tag{3}$$

Substituting this value for V into equation (2):

$$f_{1-2} = \frac{g\,D\,(p_1 - p_2)}{2L_{1-2}\,G^2\,v_1} \tag{4}$$

This equation can be used to determine friction factors from test data, or it can be used to predict the rate of pressure drop for a given flow rate if the friction factor is known.

In general the friction factor for this single phase flow is a function of the Reynolds number and tube roughness. The Reynolds number is given by:

$$Re_{1-2} = \frac{V_1\,D}{\mu_1\,v_1} \tag{5}$$

Substituting for V from equation (3) gives:

$$Re_{1-2} = \frac{G\ D}{\mu_1} \qquad (6)$$

The seond process in the tube is of a much more complicated nature, being a flashing flow of liquid and vapor from point 2 to point 3, at the tube exit. Writing the steady flow energy equation between these two points:

$$Q_{2-3} = (h_3 + \frac{V_3^{\,2}}{2g\ J} + \frac{Z_3}{J}) - (h_2 + \frac{V_2^{\,2}}{2g\ J} + \frac{Z_2}{J}) + W_{x_{2-3}} \qquad (7)$$

For an adiabatic flow process the heat transferred between 2 and 3 is zero, and since there is no shaft work and the tube is horizontal, W_x is zero, and Z is constant. Equation (7) reduces to:

$$h_3 + \frac{V_3^{\,2}}{2g\ J} = h_2 + \frac{V_2^{\,2}}{2g\ J} \qquad (8)$$

Again substituting the value of velocity from equation (3):

$$h_3 + \frac{G^2\ v_3^{\,2}}{2g\ J} = h_2 + \frac{G^2\ v_2^{\,2}}{2g\ J} \qquad (9)$$

Since enthalpy and specific volume are extensive properties, and noting that at point 2 the refrigerant is still in the liquid state:

$$\left[h_{f_3} + x_3\ h_{fg_3} - h_{f_2}\right] + \frac{G^2}{2g\ J}\left[(v_{f_3} + x_3\ v_{fg_3})^2 - v_{f_2}^{\,2}\right] = 0 \qquad (10)$$

This is a quadratic equation in x_3 which may be rearranged as follows:

$$x_3^{\,2} + \left[\frac{2v_{f_3}}{v_{fg_3}} + \frac{2gJ\ h_{fg_3}}{G\ \ v_{fg_3}^{\,2}}\right]x_3 - \frac{2gJ\ (h_{f_2} - h_{f_3})}{G\ \ v_{fg_3}^{\,2}} - \frac{v_{f2}^{\,2} - v_{f_3}^{\,2}}{v_{fg_3}^{\,2}} = 0 \qquad (11)$$

For a given flow rate and set of saturated liquid conditions

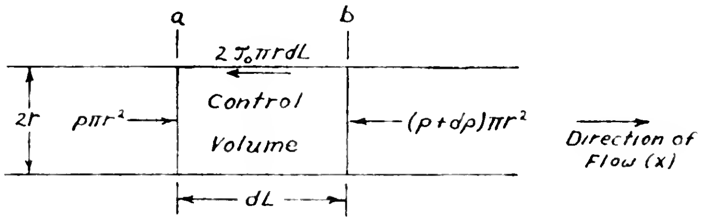
at point 2 (the temperature at point 2 is the same as at tube entrance), the quality at the tube exit (x_3) may be exactly determined by knowing the temperature at this point, and using the tabulated values of the thermodynamic properties of the refrigerant. It is well to note here that the steady flow energy equation can be equally well written between point 2 and any other point along the tube in the direction of flow, and the same equation arrived at for determining the quality. The last term on the left hand side of this equation was found to be of the order of 1×10^{-5} or less, and may be dropped from the equation.

With x_3 determined, h_3, v_3, and s_3 may be readily found by using

$$h_3 = h_{f_3} + x_3 h_{fg_3} \; ; \quad v_3 = v_{f_3} + x_3 v_{fg_3} \; ; \quad \text{etc.}$$

Also, if temperatures are known at intermediate points between 2 and 3, similar calculations can be made for these points and the thermodynamic state path of the process determined.

The flow process in section 2-3 of the capillary tube is a compressible flow of liquid and vapor. To develop an equation for this flow process, the momentum equation is applied. Consider the differential element shown below, taken at some point along the tube between points 2 and 3, in which the flashing mixture of vapor and liquid is flowing.



The momentum equation states that:

$$F_x = \frac{d}{dt}(m V_x)_{\substack{\text{control}\\ \text{volume}}} + \int V_x \, dw_{out} - \int V_x \, dw_{in}$$

For steady flow the first term on the right hand side of this equation is zero, that is, the momentum contained within the control volume is constant with time in a steady flow process. The momentum equation may then be written as:

$$p\pi r^2 - (p + dp)\,\pi r^2 - 2\,\tau_o \pi r dL = \rho_b V_b^{\,2} dA - \rho_a V_a^{\,2} dA \qquad (12)$$

The friction factor is defined as:

$$f = \frac{2\,\tau_o}{\rho V^2}$$

Substituting this value into (12) gives:

$$-\pi r^2 dp - 2\pi r f \frac{\rho}{2} V^2 dL = \rho_b V_b^{\,2} dA - \rho_a V_a^{\,2} dA \qquad (13)$$

Using the value of velocity given by the continuity equation (3), and noting that $\rho = \frac{1}{vg}$:

$$-\pi r^2 dp - \pi r f \frac{G^2}{g} v dL = \frac{G^2}{g} v_b dA - \frac{G^2}{g} v_a dA \qquad (14)$$

Integrating equation (14) between the end points 2 and 3 and simplifying:

$$\frac{g}{G^2}(p_2 - p_3) - \frac{f_{2-3}}{r}\int_2^3 v dL = v_3 - v_2 \qquad (15)$$

To determine friction factors from test data, the integral on the left hand side of this equation must be evaluated graphically. That is, the quality is determined at short intervals along the tube between points 2 and 3 by means of equation (11). The corresponding specific volumes are then readily calculated and plotted against length from the flash-off point, and the area under the curve obtained by

means of a planimeter. With the integral thus determined, the friction factor for this section of the tube may be readily calculated.

If the pressure drop due only to momentum change for this section of the tube is desired, it may be computed by dropping the friction term from the above equation, that is:

$$\Delta P_m = \frac{G^2}{g} (v_3 - v_2) \tag{15a}$$

The pressure drop due to friction in the tube may now be found by subtracting the pressure drop above from the total pressure drop through the tube.

$$\Delta P_f = (P_1 - P_3) - \Delta P_m = \Delta P_t - \Delta P_m \tag{16}$$

The integrated average specific volume for section 2-3 of the tube may be obtained from the plot described above. Since the specific volume for section 1-2 of the tube is constant, the average specific volume for the entire capillary tube is given by:

$$v_{av.} = \frac{L_{1-2}}{L} (v_1) + \frac{L_{2-3}}{L} (v_{2-3}) \tag{17}$$

The average friction factor for the entire capillary is now found from equation (4) in the following form:

$$f_{av.} = \frac{g\ D\ \Delta P_f}{2\ L\ G^2\ v_{av}} \tag{4a}$$

The average friction factor for the entire capillary tube was found to be approximately that indicated from a plot of Reynolds number versus friction factor for drawn brass tubing. This plot, taken from Technical Paper No. 409 of the Crane Company, Chicago, Illinois, is presented in Figure 10 on the following page.

The nomenclature for the symbols used in the equations herein derived is also presented on the following pages.

A complete set of calculations for test run No. 15 have been carried out in the Sample Calculations, and illustrates the use of these equations.

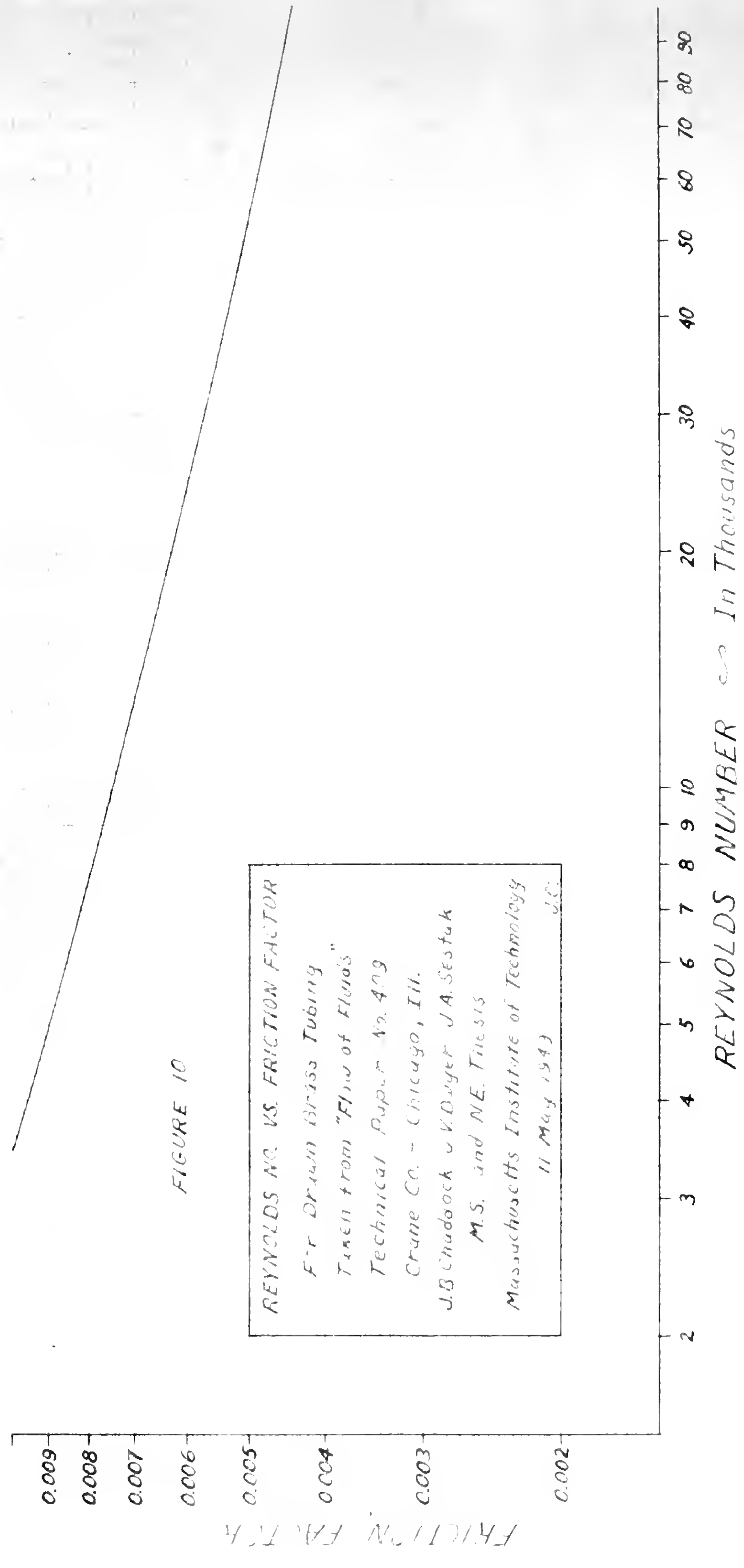


FIGURE 10
REYNOLDS NO. VS. FRICTION FACTOR
For Drawn Brass Tubing
Taken from "Flow of Fluids"
Technical Paper No. 409
Crane Co. - Chicago, Ill.
J.B. Chaddock J.V. Dwyer J.A. Sestak
M.S. and N.E. Thesis
Massachusetts Institute of Technology
11 May 1949
J.C.
FRICTION FACTOR
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
REYNOLDS NUMBER — In Thousands
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90

NOMENCLATURE

A - cross sectional area of capillary tube, ft.^{2}

d - differential

D - capillary tube diameter, ft.

f - friction factor

F_x - net force in the x-direction, lbs.

g - acceleration of gravity, 32.17 ft./sec./sec.

G - mass velocity, lb./ft.^{2} - sec.

h - enthalpy, Btu/lb.

h_f - enthalpy of saturated liquid, Btu/lb.

h_{fg} - enthalpy of vaporization, Btu/lb.

J - mechanical equivalent of heat, 778.26 ft.-lb./Btu

L - capillary tube length, ft.

m - mass, slugs

p - pressure, lbs./ft.^{2}

Δp_m - pressure drop due to momentum changes, lbs/ft.^{2}

Δp_f - pressure drop due to friction, lbs./ft.^{2}

Δp_t - total pressure drop, lbs./ft.^{2}

ρ - mass density, slugs/ft.^{3}

Q - heat transfer, Btu/lb.

r - capillary tube radius, ft.

Re - Reynolds number

s - entropy, Btu/lb.-°F

s_f - entropy of saturated liquid, Btu/lb.-°F

s_{fg} - entropy of vaporization, Btu/lb.-°F

t - time, sec.

τ_o - shear stress at the tube wall, lbs./ft.^{2}

μ - viscosity, lb./ft.-sec.

v - specific volume, ft.^{3}/lb.

v_f - specific volume of saturated liquid, ft.^{3}/lb.

v_{fg} - specific volume of vaporization, ft.^{3}/lb.

V - velocity, ft./sec.

V_x - velocity in x-direction, ft./sec.

w_r - mass rate of flow, lb./sec.

w_{in} - mass rate of flow into the control volume, slugs/sec.

W_x - shaft work, Btu/lb.

x - quality, lb. of vapor/lb. of mixture

z - height, ft.

Subscripts 1, 2, and 3 refer to tube entrance, flash-off point, and tube exit respectively.

ORIGINAL DATA

The data obtained in accordance with the method outlined in "Details of Procedure" were recorded in a computation notebook and on hectographed forms. The notebook and forms are in the possession of Professor A. L. Hesselschwerdt of the Mechanical Engineering Department.

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